FACILITATING LESSON 4

Traveling in Time

How long ago was that?

Synopsis

Students place dates of birth and historical events on number lines. Using the number lines and mental math, they add and subtract using 10's and 1's to calculate people's ages and calculate how long ago their birthdays occurred. Students record their strategies both on the number line, a tool that makes mental math strategies visible, and by writing **equations**.

1. Working as a group and then in pairs, students figure out different time spans and explain their strategies. They use the number line as a tool for computing and for showing their problem-solving methods and record their strategies using equations.
2. Students share what they learned about using number lines in order to solve problems.
3. Students examine quantities on both sides of the equal sign.

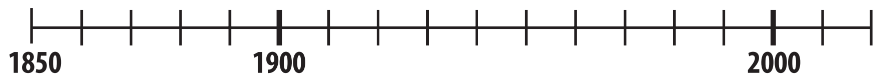
Objectives

- Use a number line to compute and to explain mental math strategies
- Use counting up and down by 10's and 1's to solve addition and subtraction problems
- Record mental math and number-line actions using equations
- Make generalizations about how addition and subtraction behave in equations

Materials/Prep

- Adding or receipt tape or strips of easel paper for number lines
- Colored markers
- Easel Pad
- Rulers
- Assorted items that have number lines—a thermometer, rulers, yardsticks, meter sticks, pictures of gauges, etc. for the *Opening Discussion.*

Make a current timeline from adding machine tape or long strips of paper, marked from 1850–2020, to post in the front of the classroom for *Activity 1*.



Make one copy of *Blackline Master 5*: *U.S. Historical Events* for each group of three students with each copy cut into a set of six cards and placed in an envelope for *Activity 2*.

Opening Discussion

Review what students remember about the number lines used in *Lesson 3*. Distribute items that use number lines (thermometers, barometers, tape measures, yard sticks, rulers, meter sticks, beakers, timelines, etc.) to pairs of students.

Say to students:

Describe the number line in your object. What does it measure?

Compare the objects, highlighting what they are designed to measure, the numbers listed, and the values of the marked intervals.

Conclude by saying:

Today we will use number lines to explain how we use mental math to find the difference between two numbers.

Activity 1: Birthday Numbers

Start with the following scenario to introduce the first problem in *Activity 1: Birthday Numbers* (*Student Book*, p. 64)

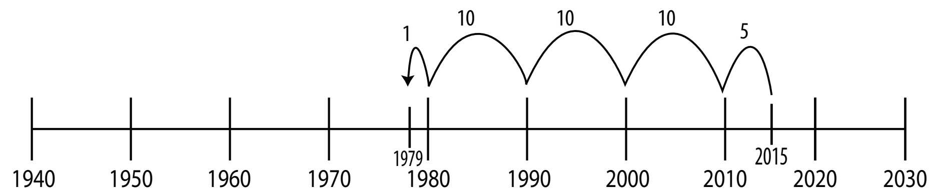
A friend of mine just celebrated her 46th birthday. I was trying to figure out in my head when she was born. In what year was my friend born? How do you know?

Invite students to share their strategies for solving the problem. Someone may be 46 years old and know the answer immediately. However, try to bring out the mental calculations students used.

Heads Up!

The following examples are written for the year 2015. Adjust the lesson to the current year and follow the same method.

Starting with the first volunteer, write on the board the mental steps he or she used to determine the birth year for the 36-year-old. Use the number line to demonstrate the steps, explaining the student's process as you write. For instance:



You started at this year (2015 or the current year, 20XX). You went back 5 years (or X years) to the year 2010, a round number. Then you went back 30 years to 1970, and then you went back one more year to 1969. Adding 5 and 30 and 1, you get 36 years.

Did anyone solve the problem a different way?

Listen to examples, tracking students' reasoning on the number line. If student examples are not forthcoming, offer one of your own.

Once you record the number-line jumps and written steps for a few strategies on the timeline, begin to describe the steps, using mathematical notation. For instance, for the example above, you might write and talk through the problem as follows:

$2015 - 15 = 2000$

$2000 - 30 = 1970$

$1970 - 1 = 1969$

$2005 - 15 - 30 - 1 = 1969$

Ask students to write both the number-line jumps and an equation in their books as they complete the problems on the page.

Take time for students to share number-line strategies and equations for each problem.

Name the methods you see students using—counting back by 10's, or multiples of 10's; adding on by 1's and 10's; or finding a benchmark date, such as 2000, and working from there. Summarize briefly:

All these strategies help us find the difference between two dates and how far apart different birth dates are from the current year, 2015 (or 20XX).

The equations show—in writing—how we add and subtract mentally. Though it might take more steps to calculate in our heads than with paper and pencil, what are some of the advantages of calculating mentally?

Allow time for learners to complete *Activity 1* and briefly discuss strategies used.

Activity 2: How Long Ago?

Distribute one envelope of cards cut from *Blackline Master 5: U.S. Historical Events* to each group. Tell students:

History sometimes seems like a jumble of dates and events. We often wonder, "When did that war happen?" or "How long ago was that?"

You have a set of cards depicting important events in U.S. history. With books closed, please arrange the cards in the order in which you think the events happened, starting with the one farthest back in time and working up to the most recent one.

Then look at your book to find the date of each event. Write the dates on your cards and rearrange them, if necessary.

After students have arranged the cards, review the directions for *Activity 2: How Long Ago?* (*Student Book,* p. 66). Clarify the term "difference," then ask students to answer the questions, and as you circulate, notice who is working with 10's and 1's comfortably. Always ask:

How would you find the answer in your head without using pencil and paper?

What is another way you could do the problem?

Notice how students determine missing numbers. Do they count spaces accurately? Do they see the patterns of increments? Notice how they use the number line. Do they mark off 10's and 1's accurately? Do they use multiples of 10? The more fluent the students become, the more likely they will be able to work with larger increments of numbers, preferring to go down by 50 rather than 10, for instance. Encourage this efficiency and connect it with five 10's jumps on the number line when sharing strategies with the class.

If no one chunks time into larger intervals, ask them how they could do that.

Ask students to share their methods by putting their number lines and equations on easel paper. Post these for the class to see.

Choose some examples and ask:

How did you know what the missing numbers were here?

What other numbers did you put on the number line? How did you decide where to place them?

Compare methods students used. Then focus on their solutions, asking:

Did you do the problem first in your head or on the number line?

Did counting with 10's (or multiples of 10) help you? How so?

Then focus on their solutions, asking:

How did you know how many years ago this event happened? How do you know that the event happened more or less than 100 years ago?

If students subtract in some other way (traditional algorithm, for example), make sure they also show what they did on the number line.

Because student pairs will only choose two or three events, you may want to assign them the rest for homework as extra practice.

Math Inspection: Make It True

This math inspection extends the work with the equal sign by including the subtraction sign along with the equal sign and addition sign. As they explore the relationship between the numbers, students should begin to realize that addition and subtraction are related; in fact, they are inverses of one another. Two of the final equations will illustrate the idea of additive inverse (**40 + 20 – 40 = 20** and **40 = 20 + 40 – 20**). Students should check to see that their strategies work for any pair of numbers, *a* and *b*. If they are only dealing with positive numbers, they may say that the first number has to be the larger.

This inspection deals with the reflexive property of equality:

$a + b = a + b$ or $a - b = a - b$

Anything is equal to itself.

This idea shows up in algebra: In the equation $a + b - a = b$, on the left side of the equal sign, adding *a* and then subtracting a results in 0, so b is left on both sides of the equation. The same is true if the action happens on the other side of the equal sign: $a = b + a - b$. On the right side of the equal sign, addition and then subtraction of b results in 0, so a is left on both sides of the equation.

Ask students to work individually to determine the solution to each problem.

Then, as pairs, have them share how they determined where to place addition and subtraction signs. Post the different solutions and ask students to articulate the similarities and differences among them. Give students time to decide if this will work with any pair of numbers, and ask for examples. Finally (#4), ask:

Why will equations patterned like 40 + 20 – 40 = 20 and 40 = 20 + 40 – 20 always work?

You may hear comments like:

> "Because you are adding and subtracting the same amounts."
>
> "Because adding the 20 cancels subtracting 20."

If no one brings it up, record this in algebraic notation: $a + b - a = b$

This is the moment to have students check to see that their strategies work for any pair of numbers, *a* and *b*. If they are only dealing with positive numbers, they may say that the first number has to be the larger. If students are familiar with negative numbers, volunteers may offer examples such as -25 + -60 – (-25) = -60. Acknowledge that yes, the generalization works for more than just positive whole numbers. If this does not come up, you can raise it at this point or return to this point in *Lesson 6: Extending the Line,* after students have had more experience combining positive and negative numbers.

Math Inspection: Check Both Sides of the Equal Sign

This inspection extends the work with the equal sign and asks students to notice patterns that occur with subtraction. This is a case in which noticing patterns and generalizing from them can lead to important insights about operations and the mathematical concept of equality.

In the equations for students to consider (for example, 9 – 6 =10 – 7), they are asked to look at the relationship between the numbers on the left and right of the equal sign. To maintain equality, each number increases by one. The relationship among the numbers remains stable and so the answer does not change. This is different from what we see with addition (for example, 5 + 2 = 6 + 1). With addition, if one number on one side of the equal sign increases, the other on that same side has to decrease for the sum to remain the same. Students may be confused or surprised by this contrast at first. Use number lines to illustrate what is happening in specific equations. Students may find it helpful if you and they mark off steps down a hall or across the classroom so they can act out sample equations with numbers less than 20. In this way they have a kinesthetic means to experience what changes and what stays the same.

As you prepare, you may notice that in the subtraction problem on the right (10 – 7), the numbers are each 1 more than the subtraction problem on the left (9 – 6), but the difference is the still the same (3). In the second equation, each number is three more. In the third, each number is 25 more. The difference remains the same as the difference between the original numbers. Another observation might be that in each case, the expression on the right side of the equation is easier to simplify since it is subtracting a multiple of 10, and that the right side can be created from the left by adding the same number to each value (for instance adding a 1 to the 9 and to the 6 results in 10 – 7). In other words, this is a good strategy for making numbers friendlier in order to do mental math calculations or to eliminate the need to borrow.

In mathematical terms, this is the zero identity property at work. For instance, 48 – 13 = (48 + 2) – (13 + 2), which can be rewritten as 48 + 2 – 13 – 2, and + 2 and – 2 equal 0.

Begin by asking students to articulate again what the equal sign means (that the value on the left is equivalent to the value on the right). Ask them to verify that this is true for each of the three equations.

Now write **9 – 6** on the board.

Then write **10 – 7**.

Ask:

Are these amounts equal? (Yes)

Then ask:

How do you know?

People may have varied responses (for example, because I know my facts—they are both 3). Record responses on the board for all to see.

If no one says that each amount increases by 1, ask:

How do the amounts on the left change compared to the amounts on the right side?

Focus on the idea that "10 is 1 more than 9 and 7 is one more than 6, yet you still have the same difference."

Ask pairs to work together on problems 1 through 3, using this idea. As you circulate, you might ask students to think about how to show (with chips, pennies, on a number line) why this works with subtraction. Ask them to compare with their generalizations in the *Math Inspection* in *Lesson 2*, where 9 + 6 = 10 + 5.

Bring the class together to share what they have noticed.

Instruct students to put their pencils down. Talk through Problem 4. Ask them to discuss their mental math strategies for determining each of the answers.

You or students may summarize the take-home point, which is: If you are looking for the difference between two numbers, you can add the same amount to both numbers and not change the difference. Make sure everyone is clear, and if not, allow more time to demonstrate and restate this concept.

Summary Discussion

Everyday Numbers
Mental Math
3(4 + 2)
Visual Models

Ask students to turn to a partner and share one new idea they learned in this lesson.

End this conversation by asking students what they learned about comparing two amounts:

Think about all the problems you worked on in which you found the difference between dates (and times). What did you learn that helped you solve those problems?

If the following points are not raised, bring them up yourself:

- A number line can help you understand and solve problems.
- Counting up or down by 10's can help you solve an addition or subtraction problem more easily.
- There is more than one way to solve problems; sometimes adding on is easier and other times counting back is easier.
- Any time you have two or more quantities that are different in number or size, you can use a number line to compare them.

Suggest that students record vocabulary definitions (*Student Book*, p. 254) and respond to the reflection questions (*Student Book*, p. 259).

Practice

Use Up the Space—Plan Well, p. 75
For practice marking even increments on number lines.

On the Number Line, p. 76
For more practice locating numbers on the number line and using them to solve problems.

Mental Math Practice

Count Up and Down by 10's, p. 77
For skip counting practice.

By What Did I Count? p. 78
For more practice with skip counting.

Extension

Life Line, p. 79
For practice making a number line with personal life events.

Test Practice

Test Practice, p. 80

Looking Closely

Observe whether students are better able to

Use a number line to compute and explain mental math strategies

Emphasize that a number line helps one think about the meaning of the problem, rather than only the question "Do I add or subtract the two numbers?"

Do students understand equal intervals? Make the connection between equal-sized spaces and the number of years each space represents. If students are unsure about how to label amounts between marked intervals, ask where the halfway mark is and what number that would be.

Are students able to set up the number line? Help them move from the smallest number on the left to the largest number on the right; choose a starting point appropriate to the problem; recognize when they need to label each number; label the jumps (for instance, +10, +3); and then use those labels to arrive at the solution of the problem, as well as to write their equations.

Help students get started if they seem confused. For example, for Problem 3 in *Activity 1*, tell students to start with the year 1949. If they add 10, what number do they have? How many more numbers must they add to get to the year 1999? to 2000? to 2015?

If students use the number line but only write the starting and ending numbers and do not indicate any moves other than the jump, ask, “How do you know how many years that is?” Help them by starting the count yourself.

Use counting up and down by 10’s and 1’s to solve addition and subtraction problems

Are students counting by 10’s and multiples of 10? Encourage them by repeatedly asking them to do so in each problem.

Draw students’ attention to the year 2000, and ask them why that year is a good reference point. Encourage them to move first to a number that ends in zero, so, for example, if they are finding how many years have passed since President Kennedy’s assassination (1963), they can move seven on the number line to reach 1970 and continue to add years from that date.

As needed, simplify numbers so students can easily count up or back by 10’s or multiples of 10. If the numbers are too easy, adjust them so students will be challenged, starting with dates such as 1919 or 1957.

Record mental math and number-line actions using equations

Are students writing equations that accurately reflect their mental strategies and movements on the number line? Model writing ***number sentences*** and equations for the jumps they make on the number line.

How flexible are students with notation? Can they move between an addition statement and a subtraction statement and connect the two? If more than one operation per line is too confusing, advise students to limit their equations to one step and operation at a time.

Are they able to judge the accuracy of an equation? When students write their equations, suggest they check back with the moves on the number line to confirm that what they have written reflects the moves they made.

Make generalizations about how addition and subtraction behave in equations

Are students beginning to notice inverse relationships? If you add and subtract the same amount, the operations cancel each other out. Are students able to explain, in their own words, why adding the same amount to the minuend and the subtrahend preserves the difference between the numbers? Look for students to connect this to a visual model, showing the distance between two numbers on the number line remains stable when the same amount is added to elements of an equation or expression.

If students did not fully grasp this point while completing the *Math Inspection: Make it True*, they will return to inverse relationships later with a discussion of the additive inverse and zero pairs in *Lesson 7*.

Rationale

Time and time intervals play an important role in our lives. Dates provide landmarks that we use as reference points. For example, the number of days, months, or years since we no longer had to wear braces, lived at home, went on our last vacation, or got our first cell phone. There are many ways that people count these time intervals, and some claim they cannot do this without a paper and pencil. Using mental math strategies, accompanied by a visual tool (number line) to show the strategies, people can find these intervals and explain their reasoning to others.

Timelines are used to show the sequence of historical events by displaying the dates of those events as marks on the line. Number lines are sequential; the numbers increase to the right of 0 and decrease to the left of 0. Sometimes the lines include marks for every number; other times, for example, for every 10, 25, 50, or 100 intervals. Sometimes the lines include 0; other times they do not. The better able students are to count by 10's and multiples of 10, the better they will be able to travel along the number line.

The ability to calculate differences or to compare amounts mentally empowers people to make decisions more efficiently. Some may not possess the strategy of counting by 10's or multiples of 10 to reach quick solutions to problems. This lesson encourages students to build up to or down from numbers in increments of 10 or multiples of 10 to develop and refine their number sense and, thereby, their facility with numbers. The use and construction of number lines paves the way for reading and constructing graphs. This topic is developed in EMPower's data and algebra books.

Math Background

These problems about number comparisons call for either subtraction or addition to solve them:

- I'm 54, Rachel is 27, how much older am I?
- Sally lives on 53rd Street and I live on 120th Street; how far apart do we live?
- How long ago did the Wright Brothers fly their first plane?

The problems above represent two models for subtraction: (1) how much larger one number is than another (the age example) and (2) the distance between two numbers (the address and airplane examples). In each one, subtraction solves the problem. So does addition. Traditionally, school books teach these as 54 – 27 or 120 – 53. However, when people solve these mentally, they often use addition. Using the age problem as an example, people may count on:

"27 to 30 is 3, to 54 is 24, and 3 and 24 is 27."

Or they count on and adjust:

"27 to 57 is 30, but I've gone too far, so it's 3 less, or 27."

And when subtracting, people sometimes explain the problem as follows:

> "54 minus 4 is 50, minus 20 is 30, minus another 3 is 27, so that's 4 and 20 and 3, or 27."

Or they may picture the traditional vertical algorithm and borrowing:

$$\begin{array}{r} {}^{4}\,{}^{1} \\ 54 \\ -27 \\ \hline 27 \end{array}$$

Here is another way to notate this:

$$\begin{array}{r} 40 \\ -20 \\ \hline 20 \end{array} \qquad \begin{array}{r} 14 \\ -\ 7 \\ \hline 7 \end{array} \qquad 20 + 7 = 27$$

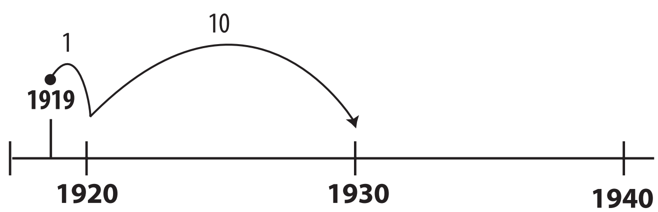
All of these strategies get to the solution, in this case 27. The focus in this lesson is on counting forward or back and using the number line to help show the moves.

Using groups of 10's, 5's and 1's to count is one of the first efficient methods that students acquire for totaling or comparing numbers. In this lesson, you may notice that some students count only by 1's. The number line can encourage different strategies, such as counting on, adding on, and counting back.

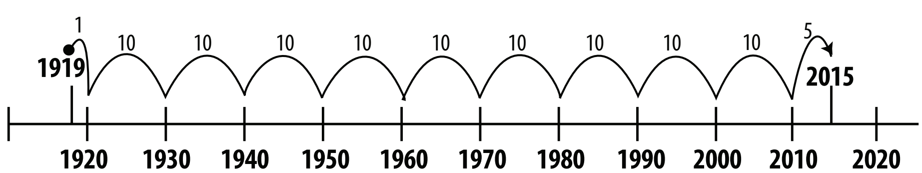
The following is one way you might introduce those categories in class.

I drew a number line to show when Prohibition started.

I went from 1919 to '20, '20 to '30, and then what?

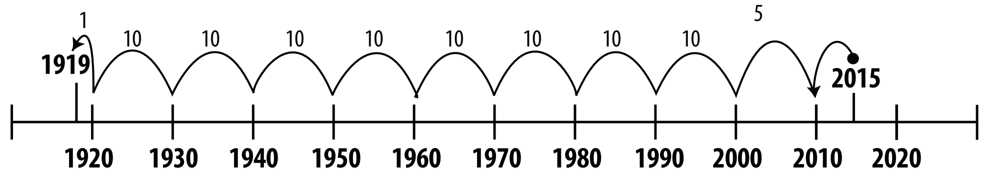


As students answer, for instance, '30 to '40, and '40 to '50, add these to your number line, and indicate the number above each jump, like this:



This way is called "count on or add up to a multiple of 10."

What if you started with the larger number and counted back? I started at '05, then went to '04, '03, '02, '01, '00, then '90 then '80... then what?



As students tell you the rest (for instance, '70, '60, '50, … '20, '19), ask them to help you label the numbers, and write

1 + 1 + 1 + 1 + 1 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 1 = 96 years

or

5 + 90 + 1 = 96 years.

This way is called "subtract/count down to a multiple of 10."

Working with 10's and 1's reinforces the place value system we use for numbers. As students become more proficient, however, expect that they will count by multiples of 10—by 20's, 50's, or 100's.

Context

Most of the historical dates in this lesson center on 20th century U.S. history. If students in the class are from other countries, ask for important dates in their own countries' histories, locate them on the timeline, and do some math with those dates.

Facilitation

Acknowledge students' interest in, and history with, traditional algorithms but remind them that mental math is extremely useful in daily life and will also strengthen their ability to select the correct answers on tests. Often on tests, they are asked to solve a problem more than one way; they can use their algorithm for their second way.

Some students will need to label the number lines in more detail than others. Encourage them to do that and then move them to bigger intervals. That is, when finding the number of years between 1939 and 2005, some will mark only 1939, 1940, and 2005, and note their jumps, although others will need to mark more numbers between those dates.

Making the Lesson Easier

Together, mark off 1's on at least part of the number line.

Practice counting by 10's , starting at a number between decade numbers—at 1933 or 1987, for example.

Refer students to *Mental Math Practices: Count Up and Down by 10's* (*Student Book,* p. 77) and *What Did I Count By?* (*Student Book,* p. 78), where they practice counting back and subtracting.

Consider working only from events with easy dates such as 1920, women getting the vote; 1935, The Great Depression; or 1945, the end of World War II.

Find historical events that are closer to the current date or, as a jumping-off point for comparing numbers on a number line, have students work from their birth dates (in the 1900's), rather than the current date.

Ask:

How many years ago were you born?

Making the Lesson Harder

Choose dates farther back in time, thus using 100's as well as 10's and 1's: Simon Bolivar, South America's "George Washington," declares Venezuela's independence from Spain (1810); the United States Declaration of Independence (1776); the Discovery of Photosynthesis (1779); the Discovery of Bacteria (1683); Queen Elizabeth I ascends the throne in England, and Shakespeare is actively writing (1558); Christopher Columbus's arrival in the Western Hemisphere (1492); Joan of Arc leads the French army to victory over the invading English (1429); and the Norman Conquest of Britain (1066).

This teacher opened the session by connecting the context to students' lives. Here are the observer's notes from that session.

The teacher opened by saying, "I know we have some people from Haiti here, so I want to congratulate them on the 200th anniversary of their country's independence. For the rest of you who aren't from Haiti, when was that?"

Silence.

The teacher wrote "2004"on the board and asked again, "If Haiti won it's independence 200 years ago, when was that?"

Mabel, an ESOL student, replied, "Nineteen thousand."

The teacher wrote "19,000" on the board and asked, "Is this what you mean?"

Immediately, Mabel answered, "No, nineteen hundred."

"Hmm. . . Is that exact or an estimate?" asked the teacher, as she wrote "1900" to the right of the two numbers, 2004 and 2000.

"It looks like a pretty good estimate to me," declared Carol, who sat one seat over from Mabel.

"Yes, but what would it be exactly?"

Silence.

The observer asked, "What date would be 100 years ago from the year 2000?"

The students responded with a chorus of "1900."

The teacher drew a line connecting 1900 and 2000, saying, "So, one hundred years ago from the year 2000 would be 1900."

"So, what would be 100 years ago from 2004?" the observer asked.

Finally, Mable announced, "1904."

The teacher wrote "1904" on the board. "That would be exactly 100 years ago from this year. What would be 200 years ago?"

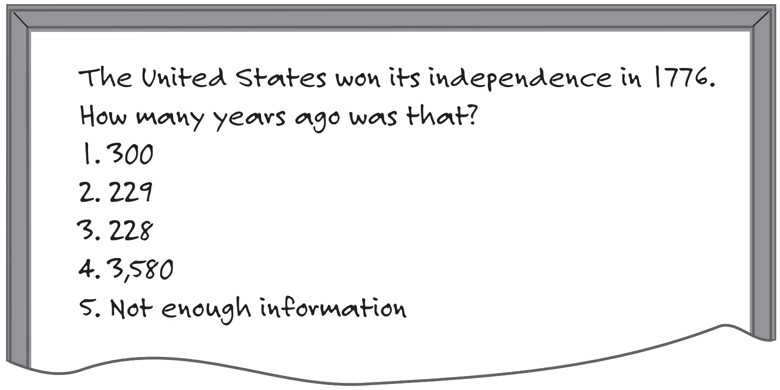
Several students called out, "1804," which the teacher wrote on the board.

The teacher then provided the historical date 1776 and asked how many years ago that had been. When no one replied, she explained, " You need to know this date for citizenship tests and for the GED. The United States won its independence in 1776. How many years ago was that?"

continued on next page

continued from previous page

Students forwarded several guesses, and the teacher posted these as if they were answers for a multiple-choice question. She wrote on the board:



The teacher asked for a show of hands as to which answer students thought was right. Everyone chose 228.

"How can we check that to see if it's right?" she inquired.

"Add them," came the reply from several students.

The teacher set up the problem, and a student performed the calculations. They were all correct.

Esther Leonelli, observed by Tricia Donovan
Notre Dame Education Center, Boston, MA

[The students] had good understanding of the use of number lines and round numbers to help estimate and calculate exact time periods. I adapted this lesson for higher level students using a Sierra Club leaflet showing how old the giant redwoods are (3,500 years!)

During class I prompted the students to confer with classmates about the way to solve a problem. Comments included, "You could do this (calculating the number of years that have passed since the Vietnam War) a lot of different ways," and, "How can I tell which happened first?" I asked students: "What other 'jumps' could you make on the timeline?"and "When the time period on the timeline is greater, what happens with the 'friendly' or rounded numbers used on the timeline?"

Adam Weiss
Bronx Adult Learning Center, New York City, NY

Blackline Master 5: U. S. Historical Events

President John F. Kennedy Assassinated

Photo courtesy Library of Congress Prints and Photographs Division
Washington, D.C. 20540 USA LC-USZ62-117124

United States Civil War Starts

Photo courtesy Library of Congress Prints and Photographs Division
Washington, D.C. 20540 USA

World War II Ends

Photo courtesy of Veterans History Project, American Folklife Center,
Library of Congress, August, 1945. Photo by Robert Lee Olen.

Civil Rights Act Passed

Photo courtesy Library of Congress, Prints & Photographs Division,
U.S. News & World Report Magazine Collection, LC-USZ62-95480 (9-12)

American Women Are Granted the Right to Vote

Photo courtesy Library of Congress Prints and Photographs Division
Washington, D.C. 20540 USA LC-USZ62-75334

The Vietnam War Ends