Integrating Language While Teaching Mathematics

NCELA Teaching Practice Brief

Practice Brief on Effective Instructional Practices and Examples for Teaching Math to English Learners

This is the third publication in a series of briefs that are designed to highlight promising practices for educators and other staff who support English learners (ELs) and their families with English language acquisition and the maintenance of native languages.

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Introduction

This is the third NCELA publication in a series of briefs designed to highlight promising practices for educators who teach English learners (ELs) and who assist their families with English language acquisition.

This practice brief provides educators with current research findings and evidence-based, high quality, instructional mathematics' practices from the Institute of Education Sciences' Practice Guide for teaching English learners (ELs) academic content (Baker et al., 2014) and highlights instructional practices used to teach ELs according to the National Academies of Science, Engineering, and Medicine (2017).

Teachers may find this brief helpful in explaining how research findings can be incorporated into classroom practice. It also presents examples and useful resources to prompt instructional improvement.

Educators may find this brief helpful in building a better understanding of what quality instruction includes to facilitate meaningful changes to their teacher training practice.

In addition, the brief provides educators who train teachers, with the latest research on how to better prepare the next generation of teachers. Five key teacher educator and leader practices featured in this brief include:

1. Embrace asset beliefs that position and support ELs as full participants in mathematical learning
2. Engage ELs in meaningful interactions and discourse with others
3. Provide support for ELs to engage in mathematical practices
4. Sustain an explicit focus on language as it is used in math
5. Design mathematical learning experiences that engage ELs in rich communications integrating oral and written language

For each key practice, this brief:

- Describes and references the research
- Illustrates the practice in action
- Offers additional tools and resources

1. Embrace asset beliefs that position and support ELs as full participants in mathematical learning.

Description of Practice

Teacher actions strongly communicate beliefs to their students and are essential to ensure that ELs view themselves as powerful rather than powerless (Yoon, 2008). Teachers can view ELs through an empowering assets-based lens as students with multiple sources of knowledge which includes language skills and life experiences, that contribute to their learning of math and their full participation in classroom activities (Turner et al., 2016).

The assets-based lens more fully engages ELs in mathematical learning by drawing upon each EL students' experiences outside of the classroom in the everyday aspects of life. These experiences can assist educators and leaders in developing ELs' understanding of math (Civil, 2016). These rich experiences, also known as "funds of knowledge," may include EL students' personal, familial, community, and peer experiences (Gonzalez, Moll, & Amanti, 2005). Making connnections with students' day-to-day life experiences can create opportunities for them to develop comparisions to help them in their understanding of mathematical concepts (Chu & Rubel, 2013; Richland & Begolli, 2016). Teachers who incorporate ELs' assets understand the importance of integrating their students' day-to-day life experiences to promote academic success and language development.

Such assets-driven teaching (in other words teaching math based on day-to-day life experiences) runs counter to other practices that view language as a prerequisite for ELs to engage with important math concepts and practices (de Araujo, 2017). Indeed, the language used in the home can be a vital asset for ELs when the student's native language is recognized as valid and as a contributor to the student's understanding of mathematics (Moschkovich, 2013).

Practice in Action

The following example was highlighted in the recent National Academies synthesis (2018). It highlights embracing asset beliefs toward ELs and providing appropriate and flexible supports—or scaffolding—in their teaching (Bianchini, 2018). The example is set in a dual language classroom in an elementary school.

Example Scenario:

Teachers and students engaged in telling stories in the following way: "Yo fui al mercado y compré cuatro _____y los metí en una bolsa." ("I went to the supermarket and bought four _____ and I put them in a bag.") At this point, students were invited to include items they liked, in which case children chose four toy horses. The teacher continued with the story to expand on it and create the following multiplication type problem: "I went to the supermarket and bought four toy horses. The horses had four legs each. I put all four of the horses in one bag. How

many legs are in the bag?" [Students had opportunities to connect the number of legs each horse had to the problem.]

Students engaged in solving the problem using their small white boards, and the teacher called a student to communicate his solution. The student began explaining his solution by drawing four toy horses and counting each leg verbally and by pointing to each leg as he counted, and then writing an equation (4 X 4 = 16) that connected to his pictorial representation… [T]his was the first time that this particular student was making sense of or appropriating the multiplication symbol. Because not all students were using the multiplication symbol yet, the teacher used the details of the story as well as the student's explanation to scaffold for the rest of the class what the multiplication symbol meant, (i.e., 4 + 4 + 4 +4 = 16), an equation that most students were familiar with in this mathematics classroom. She pointed to the student's pictorial representation and connected each detail to the representation using repeated addition (Celedón-Pattichis, 2018).

Tools and Resources

The project known as Teachers Empowered to Advance Change (TEACH) in Mathematics provides a Community Mathematics Exploration Module that may serve as a helpful resource to assist teachers in understanding the community assets and lived experiences of their students with an explicit emphasis on helping EL students connect their life experiences to classroom mathematics (Turner et al., 2015; see https://teachmath.info/modules/community-exploration-module/). To supplement the activities contained in this module, teachers will find that connecting with their EL communities may lead to more effective teaching. Understanding the ways in which communities use languages other than English and how those languages have been integrated into local community practices will help teachers more effectively engage their EL students.

The National Council of Teachers of Mathematics has further described Asset-Based Approaches to Equitable Mathematics Education Research and Practice (https://www.researchgate.net/publication/326219362_Asset-Based_Approaches_to_Equitable_Mathematics_Education_Research_and_Practice)

2. Engage English learners in meaningful interactions and discourse with others.

Description of Practice

Meaningful interactions occur when students are engaged in extended conversation or back-and-forth discussions that build understanding (Walqui & Heritage, 2018; Chu & Hamburger, 2019). Students engage in back-and-forth discussions when they take long or multiple turns to expand upon an idea beyond just one or two words. These back-and-forth discussions become "reciprocal" when the interactions are not scripted or dominated by one speaker, and when different students respond to each other by adding to, building on, or challenging ideas. Unfortunately, many ELs do not have opportunities to engage in such meaningful interactions in English unless their teachers intentionally create environments and use engaging tasks to invite and support their full participation.

One approach to fostering meaningful interactions in math classrooms has focused on solving rich mathematical problems that invite students to share and compare strategies and approaches (Stein et al., 2009). To ensure that ELs can engage in these meaningful whole-class discussions, teachers can provide models of useful language and give students time to rehearse their responses. Under these conditions, ELs can engage in meaningful whole-class interactions.

An alternative and complementary approach involves sharing key information that one student has and the other student needs (Ellis, 2003). Communicative tasks that require ELs to bridge such "gaps" in information provide opportunities to attend to key features of mathematical objects or problems as they talk to one another (Chu & Hamburger, 2019). For example, students can engage in a class project in which different expert groups solve different problems and then share their findings (Walqui & van Lier, 2010). If this discussion is guided by a carefully selected set of focus questions, then as ELs share novel information with their peers, they will be able to identify key features that cut across a family of problems.

Practice in Action

In a 10th grade geometry class at a school for recent immigrant newcomers, students are investigating how applying a scale factor affects the surface area of geometrical solids. The teacher has organized class activities for this lesson as a "jigsaw project" (Walqui & Bunch, 2019). In a jigsaw project, different students will have different pieces that they will put together to see the "big picture" of what they are learning. Students begin in "base" groups and are assigned by counting off by numbers to expert groups where they will investigate a specific case.

In their expert groups, students apply their previous knowledge of using nets to analyze the surface area of geometric solids. Together, each expert group applies the scale factor the respective group was assigned to recalculate the surface area of the transformed object. As they compare their original and scaled surface areas, they complete a matrix of focus questions that

draw their attention to the key aspects of the problem. The focus questions target the key ideas of scale factor and surface area so that students of all English language proficiency levels have access to the ideas. Each expert group reaches a consensus on how it will report its findings concisely to its base groups. In the expert groups, students have a chance to rehearse what they will say when they return to their base groups, providing ELs with opportunities to deepen both conceptual understanding and fluency in using language to express mathematical ideas. In addition, the expert groups naturally have students with different levels of English language proficiency, students can make language choices together; they can come to a consensus on how best to use language, including the targeted mathematical terms, to report to their base groups.

Back in their base groups, students share key findings from their expert groups, and begin to compare and contrast similarities and differences across different expert groups. The teacher leads a whole-class discussion in which experts present some of their results. As a class, they reach the conclusion that surface area is scaled by a factor that is the square of the linear scale factor.

Tools and Resources

In general, jigsaw projects (like the one described in the example above) have the following steps:

1. Identify a key learning goal related to solving a variety of problems (e.g., applying slope-intercept form to solve story problems).
2. Select a variety of problems that share similarities but also differences (e.g., different given information or unknowns, or different signs and quantities for the slope and intercept).
3. Compose focus questions that will require students to think about and discuss the targeted goals (e.g., "How did you use slope and intercept in solving the problem?").

This video from the Teaching Channel (2013) gives an example of one implementation of a jigsaw project (https://www.teachingchannel.org/video/groups-to-analyze-complex-texts).

A similar example from humanities is part of the Understanding Language unit *Persuasion Across Time and Space,* focusing on three key historical texts (Walqui, Koelsch, & Schmida, 2012).

3. Provide support for English learners to engage in mathematical practices

Description of Practice

The National Council of Teachers of Mathematics (NCTM) (2000) defines mathematical practices as a process involving:

- Problem solving
- Reasoning and proof
- Connections
- Representations

Since 2010, many states have adopted college and career-ready standards which have included activities such as:

- Explain and justify work
- Make sense of problems and persevere in solving them
- Look for and make use of structure (e.g., patterns in numbers, shapes, or algorithms)
- Choose and use appropriate tools when solving a problem
- Apply mathematics to problems arising in everyday life, society, and the workplace (Opfer, Kaufman, & Thompson, 2016)

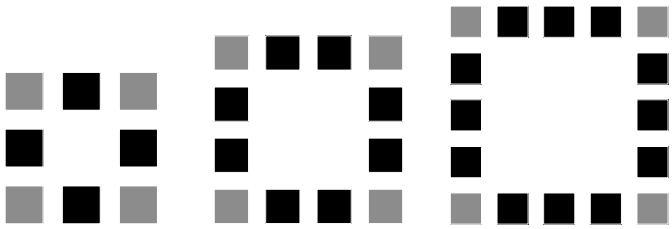
Teachers who wish to incorporate activities such as those listed above can provide ELs with supports to interact with their peers and use language as they engage in mathematical practices (Koelsch, Chu, & Bañuelos, 2014). They will need to think about multiple ways to tackle math problems also known as "finer-grained" practices. Finer-grained practices make up activities such as: understanding the problem, creating a plan, carrying out the plan, and evaluating the reasonableness of the solution. The example below shows how to implement the finer-grained practices. Additionally, these four examples of finer-grained actiovites may also be further broken down into more specific actions that students can take. For example, in understanding the problem, the students may need to identify the unknown. By giving explicit names for these actions, teachers support ELs in engaging in these activities and reflecting on their strategic choices as they become more experienced and sophisticated participants in these activities.

Teachers can offer ELs common/formulaic examples or specific models of the language they can use as they interact with peers (Koelsch, Chu, & Bañuelos, 2014). These models offer students ways to connect ideas and justify their reasoning in a supportive classroom environment, allowing ELs to develop analytic and language skills (Heritage, Walqui, & Linquanti, 2015).

Practice in Action

To assist ELs with the practice of looking for patterns, teachers can offer a "guidance card" that has specific language choices for a conversation (Chu &Hamburger, 2019). The following "guidance card" offers ELs with two kinds of choices—strategic and linguistic—as they work with a partner to engage in two related mathematical practices: 1) looking for and making use of structure; and 2) looking for regularity in repeated reasoning. First, they select a strategic action and announce it to their peers (e.g., "point out what is the same or different"). Second, they select one of the models that offer a specific way to practice (e.g., "Both of these … have…"). Students then engage in a discussion responding to and building on each others' ideas (Chu & Hamburger, 2019).

For example, a small group of students can be given cards with the following figures. As they work together to figure out the pattern and how to extend this pattern geometrically, a guidance card can support their discussions.



Structure and Regularity Guidance Card

What you can do	What you can say
Point out what is the same or different.	Both of these… have… What I look at…, this… has more/less… Unlike…, this… has…
Put into groups and take groups apart.	If I put together…, I get… If I look at…, I see… If I take apart…, I have…
Find and describe what repeats.	Something that I see repeating is… If I keep doing…, I will get… A shortcut for repeating this is…

In the dialogue below (Chu & Hamburger, 2019), the elements that students are using from the guidance card are italicized. The plain text reflects the information that students are inserting as they discuss the patterns they see in the sequence of shapes.

Student A: I am going to *point out what is the same. Both of these* squares *have* four grey squares in the corners.
Student B: *Unlike* your small square, *this* square *has more* black squares.
Student C: I want to *put into groups*. *If I look at* the sides, *I see* four groups of three.
Student A: Mine has four groups, but each has two squares.

The model expressions (e.g., "If I put together…, I get…") assist students as they connect different aspects of the geometric pattern.

Tools and Resources

Lists of Math Practices:

Links to mathematical process standards from the National Council of Teachers of Mathematics (NCTM) can be viewed on the following page: https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Process/A 2016 survey of the nationally representative American Teacher Panel (Opfer, Kaufman, & Thompson, 2016) also identified multiple mathematical practices that teachers can use to engage their students (see p. 88–89): https://www.rand.org/pubs/research_reports/RR1529-1.html

Similarly, the new framework from the 2025 National Assessment for Educational Progress includes a list of five mathematical practices: https://www.nagb.gov/news-and-events/news-releases/2019/release-20191121-governing-board-approves-updates-mathematics-framework.html

Teacher-friendly Resources:

Guidance cards for mathematical practices have elements of both "what you can do" and "what you can say" (Chu & Hamburger, 2019). One way to create a guidance card for a particular mathematical practice is:

1) Read the description of the process or practice standard.
2) Identify each practice within that description.
3) Group practices and come up with a general label (e.g., "Put into groups or take groups apart").
4) Find formulaic expressions, models, or other language that might be helpful to students.
5) Test, refine, and revise the actions and language with students and solicit their feedback.

As a further example, Koelsch and colleagues (2014) offer a graph interpretation card:

Interpreting Graphs Guidance Card

What you can do	What you can say
Attend to labels.	The title of the graph is . . .. The x-axis represents . . .. The y-axis represents . . ..
State specific values.	The maximum value of the variable . . . is . . .. The minimum value of the variable . . . is . . .. The mode of the variable . . . is . . .
Describe relationships.	When the value of . . . is . . ., . . . the value of . . . is . . .. If the value were . . . for . . ., the other variable could be . . .. As the value of . . . increases/decreases, the other variable . . ..

4. Sustain an explicit focus on language as it is used in mathematics.

Description of Practice

Across grade levels, ELs frequently encounter story problems, in which a math task is embedded within a real-world situation or context. Numerous studies have shown how such story problems may pose unintended cognitive demands and burdens on ELs (Lager, 2006; Martiniello, 2008). Teaching key words is not sufficient to support ELs, because word problems often do not explicitly name variables or quantities and instead ask questions using units (for example "How many feet of ribbon are needed?" does not name the variable *length*) (Koelsch, Chu, & Bañuelos, 2014). Therefore, teachers can support ELs understanding of story problems in three ways: 1) understanding the key elements of real-world situations; 2) naming variables; and 3) identifying relationships.

Practice in Action

For ELs, the mathematical practice of "make sense of problems and persevere in solving them" requires first understanding what the problem is asking. Drawing upon Polya's (1957) seminal and classic four-step approach for solving problems, the "Math Clarifying Bookmark" outlined below models the actions and language necessary to "understand the problem." Teachers may first demonstrate how to use this strategy to unpack story problems and introduce in context, key terms such as "unknown" and "variable." Then teachers can support ELs to engage in student discussions about problems. Working in pairs, students read a story problem together and then take turns discussing their understanding of the problem using the actions and language offered below. Over time, ELs who have had multiple opportunities to use this language with peers will be able to do so independently.

Math Clarifying Bookmark (to Make Sense of Problems)

What you can do	What you can say
Identify what the problem is asking.	The unknown in this problem is… The units of the unknown are… Reasonable values for the unknown would be…
Identify the given data and constraints.	The variables or quantities in this problem are… The values given in the problem are… This problem assumes that…
Draw a picture or model to represent the problem in a different way.	I can show this problem by… A model that represents this problem is… I can represent this part of the problem with…

Tools and Resources

The Mathematics Assessments Resource Service (n.d.) offers many rich story problems that are organized by conceptual goal and annotated to reflect typical student thinking. These problems are available in varying levels of mathematical proficiency (e.g., "Novice," "Apprentice," and "Expert"). Specifically, "Novice" problems highlight two mathematical practices only ("reason abstractly and quantitatively" and "attend to precision"), while "Apprentice" problems also require students to engage in two additional mathematical practices ("construct viable arguments and critique the reasoning of others" and "look for and make use of structure"). Even "Novice" problems may still have language demands that are challenging for reasons that are not related to the complexity of the mathematics. For this reason, ELs may benefit from linguistic scaffolds or discussions with peers to ensure they understand the contexts and non-technical vocabulary within the story problems.

In addition, the Mathematics Assessment Collaborative has an expanded library of Performance Assessment Tasks These "performance assessment tasks" offer in-depth opportunities for problem solving.

5. Design mathematical learning experiences that engage English learners in rich communications integrating oral and written language.

Description of Practice

As ELs explore and connect new math concepts, they will need many well-supported opportunities to use language in listening, speaking, reading, and writing (Baker et al., 2014). As teachers plan instruction for ELs, it is helpful to think explicitly about how the language that students use will develop. As ELs deepen their understanding and engage more deeply in math practices, their language production will grow from the more dialogic (i.e., back-and-forth conversation) to the more monologic (i.e., an edited and rehearsed speech). Lessons can support students as they grow from more tentative uses of language to explore new ideas toward using language in more polished and thoughtful ways to display their understanding (Bunch, 2014). ELs first benefit from opportunities to solve problems collaboratively, which will involve them talking (speaking and listening) as they are simultaneously writing down ideas and testing out approaches (reading and writing). Once they have reached a consensus on an approach, they will be able to engage in more monologic performances, such as individually creating a written product to show their approach and giving an oral presentation to the whole class. To produce such presentations, students must smoothly integrate oral and written language.

Lesson activities can support multiple dimensions for language growth as ELs develop greater conceptual understanding and deeper engagement in mathematical practices. Beyond the shift from the dialogic to the monologic, students can also move from using language with peers of less or equal proficiency, to becoming more skilled in use of language. They can also move from more everyday uses of language toward more technical or specialized uses of language. In this manner, the design of lessons can offer activities that support ELs' growth in language proficiency (Hamburger & Chu, 2019).

Practice in Action

Lessons that employ a Launch, Explore, Summarize structure around a single rich problem offer multiple opportunities for students to simultaneously read, listen, write, and speak (Lappan & Phillips, 2011).

In the **Launch** phase of a lesson, teachers can surface students' prior knowledge about the real-world situations and the mathematical relationships that exist within. Helpful language-rich tasks for this purpose include brainstorming tasks like "Novel Ideas Only" and opinion-raising tasks, such as, an "Extended Anticipatory Guide" (Koelsch, Chu, & Bañuelos, 2014). In a *Novel Ideas Only* task, students brainstorm by reading, writing, speaking, and listening. First, small groups brainstorm ideas in response to a prompt (such as, "When I hear the word 'balance,' I think of…"). To offer an idea, a student reads the prompt, and then offers an idea. The rest of the group is

listening, and another student echoes the idea. Then, all students in the group write down the idea so that the lists are identical. After a few minutes, the teacher leads the sharing of ideas, with the rule that students only share ideas out loud that have not already been shared aloud. As groups read their lists of ideas, the other groups are encouraged to listen carefully so that they will not repeat any ideas when it is their turn to share. Groups who have already shared are instructed to still listen carefully, and write down any ideas that they like, expanding their list.

In an Extended Anticipatory Guide task, students share their opinions about statements carefully designed to bring their prior knowledge about problem contexts to the surface. At the beginning of the lesson, pairs of students read the statements out loud using models such as, "I will read statement 1… It says… I agree/disagree with this statement because, … What do you think?" Partners both speak and listen to each other, responding appropriately, and they write down brief notes about reasons. At the end of the lesson, students revisit the statements, and discuss whether they still agree or disagree. Now, they are writing down more extended and coherent responses that cite specific evidence from lesson activities.

As ELs engage in the **Explore** phase of a lesson, they are solving problems, making connections, or developing strategies (Lappan & Phillips, 2011). To help ELs understand math problems and concepts, teachers may encourage student conversations with peers by using tools such as, the "Math Clarifying Bookmark" or "Guidance Card for Looking for and Making Use of Structure and Regularity," included above in sections 3 and 4 of this brief.

The **Summarize** phase of a lesson can offer rich opportunities for students to share their knowledge in a variety of ways. One example is a "Collaborative Poster." In this task, each student is given a different color marker and specific guidelines about the contents of a poster that represents their work. For math problems, it is critical that the contents require students to make choices, such as selecting only one of the various approaches or representations related to the problem. As students work on the poster, they are only allowed to use their assigned color. This structure uses small-group oral interactions to create a single final product (an organized and appealing poster), integrating writing to demonstrate students' understanding (Hamburger & Chu, 2019). ELs can further work with their peers to present these written products orally.

Tools and Resources

More information about these tasks is provided as part of the Understanding Language initiative's *Persuasion Across Time and Space* unit:

- Novel Ideas Only (Launch) (page 151)
- Anticipatory Guide (Launch) (page 32) (https://ell.stanford.edu/sites/default/files/ela_archives/understanding_language_materials_Jan2013.pdf#page=32)
- Collaborative Poster (Summarize) (page 159) (https://ell.stanford.edu/sites/default/files/ela_archives/understanding_language_materials_Jan2013.pdf#page=159)

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