

Introduction

This packet is a follow-up to the Toys with Wheels family literacy activities. Even if you've had to cut back on working toward your own educational goals in order to help your children keep up with their own learning, we still want to help you keep building your own skills while you're helping them.

This packet focuses on the geometry topic of circles, exploring connections between radius, diameter, and circumference. The following activities are like ones you might do in a class, starting with things you can build and do with your hands and eventually giving you confidence to answer multiple choice test-like questions. Depending on how old your children are, you may be able to do these activities with them as well. You will notice that their explorations in the Toys with Wheels activities lead to discoveries of the same circle concepts that this packet is addressing. Have fun exploring!

Materials You Will Need

- Two circular objects in your home such as a plate, round cover, bottle cap, etc.
- The small, medium, and large circles you created from the toy activity packet
- String, ribbon, or pipe cleaners

• Tape

• Ruler with inches and centimeters

Scissors

Activity 1: Investigating Circles

Using the three circles from the toy activity and the two circular objects from home, do the following:

- **STEP 1** Select one of the circles or circular objects.
- STEP 2 Cut a piece of string (ribbon, pipe cleaner, etc.) the length of the outside rim of the object you chose. Tape this string on a piece of paper and label it *circumference*.
- STEP 3 Cut another piece of string the length of the diameter of the object you chose. Tape this diameter string underneath the circumference string and label it *diameter*.





- STEP 4 Continue doing this for each of the four circles or circular objects that are left.
- STEP 5 Measure each of the five pairs of strings (circumference, diameter) to the nearest ¹/₂ inch and record in the following table.

Diameter	Circumference

Look at your table for patterns.

- How is the diameter related to the circumference?
- Where do you see this relationship in the table?
- How could you verify with your cut string lengths?
- Based on what you have observed, if you knew the length of the diameter, how could you figure out the approximate circumference? If you knew the length of the circumference, how could you roughly determine the diameter?



Take your table of values for diameter and circumference and graph on the coordinate grid that follows. Let the diameter values be the x-axis and the circumference values be the y-axis. (When you plot your data points you may have to estimate approximately where the points should be placed, as some may not be whole numbers.)



• What do you notice? How does this relate to what you discovered about the diameter and circumference from the table and with string lengths?



- Based on your graph, if the circumference was 7 inches, about what would the diameter be?
- Based on your graph, if the diameter was 4 inches, about what would the circumference be?
- Is there a way to calculate your answers without using the graph or table? Explain.
- What would remain the same and what would change in your table if you measured your diameter and circumference in centimeters? What would remain the same or change in your graph? Explain.



Activity 2: Examining More Data

A group of students gathered some data by measuring around and across circles as you did in the previous activity.

Here are their results:

Object	Diameter	Circumference	Reasonable?
Dan's can	8 1⁄2″	25 ¾″	
Patti's pie pan	9″	18 ½"	
Other cake pan	9.3″	29″	
Lid	5.5″	17″	
Paper cup	2 1⁄2″	7″	
Medium lid	5 ¾″	27 1⁄2″	

It's always a good idea to check to make sure your measurements are reasonable. For each of the entries in the table above, decide whether you think those measurements are realistic for that object. Write "yes" or "no" in the fourth column.

• Were there any measurements that you thought were unreasonable? If so, what was it about them that made you think that?

Add the data above to the graph that you made in the last activity. Use a different color or a different kind of mark (like an x or a star instead of a dot) so you can tell the added data from your original data.

• How does the graph affect your ideas about whether the measurements were reasonable?



Try to "fix" any measurements you think are unreasonable by changing one number in each pair to what you think the correct measurement might have been. Check your fixes by adding your new measurements to your graph.

• How can you tell if they are more reasonable now?

Activity 3: Circles in the Real World

Knowing how the circumference and diameter of a circle are related means you can figure out a lot of things that are hard to measure. Even though there are mathematical formulas some people use, even just knowing that the circumference is a little more than three times as long as the diameter can give you some pretty interesting estimates.

The London Eye

The London Eye is a famous ferris wheel in London, England. The circumference of the London Eye is about 375 meters! (To get an idea of how big that is, each of those carriages holds 25 people!)



Watch out for units!

If you're not familiar with meters (m), it may help you to know that 3 meters is about 10 feet.

• Estimate the diameter of the London Eye.

I think the diameter is about _____ meters because _____

How high above the water do you think the people all the way at the top are
I think they are meters above the water because
How tall do you think the buildings in the background are?
I think building A is about meters tall because
I think building B is about meters tall because
I think building C is about meters tall because
About how long are the spokes that run from the center
to the outside?
I think they are meters because



Hat Sizes

In the United States, hat size tell you how many inches it is across the inside of a hat, but it's hard to measure how many inches it is across your head! (For example, a size 7 hat is 7 inches across the inside.)



• Use a piece of string to measure the circumference of your head.

The circumference of my head is about _____ inches.

• Based on the circumference of your head, estimate how many inches across a hat would have to be to fit you comfortably. (It's okay that your head isn't perfectly circular! You're just estimating.)

I would be comfortable in a hat that is about ______ inches across.

• How did you make your estimate?

Here are some measurements that three people took and the hat sizes they bought. For each person, decide whether the hat would be too big, too small, or just right and record your answer in the table.

Name	Head Circumference	Hat Size	How does it fit?
Usha	23″	7 1⁄2	
Charlie	24 1⁄2″	6	
Liza	21″	7 7/8	

• How did you decide whether the hats would fit?



Two Bicycles

Here are pictures of two bicycles. What is similar about them? What is different?



• If the diameter of each of the tires on bicycle A is 16", what do you estimate the diameter of the tires on bicycle B is? How did you make your estimate?

• About how many inches do you think each bicycle travels when the tires make one complete rotation?

I think bicycle A travels about ______ inches because ______

I think bicycle B travels about ______ inches because ______



A Yo-Yo Trick

A yo-yo is a toy that can go up and down a string or spin at the end of it.

Before playing with a yo-yo, you have to cut the string to a good length. Many people like to have it long enough to touch the floor when they hold the top of the string at the height of their waist.



• About how long would be a good string length for you?

I would cut my string to a length of ______ because _____

To do a trick called "Around the World," you keep the yo-yo spinning at the end of the string and swing it all the way around in a circle before pulling it back to your hand. Here is a video of someone doing the trick: <u>https://www.youtube.com/watch?v=wwTtKm5EQp0</u>

• About how big a space would you need to practice this trick safely so you don't hit anything (or anyone!) with the yo-yo? How did you decide what would be safe?

• About how far would the yo-yo travel if you performed the trick once?

In the video, Paul Yath, a yo-yo champion, made his yo-yo go around 11 times!

• About how far do you think his yo-yo traveled during that trick?



Activity 4: Even Better Estimates

Through doing the first three activities, you have discovered that the circumference of a circle is about three times as big as the diameter.

- You can make good estimates for the **circumference** by multiplying the diameter by 3, but you can make even closer estimates by multiplying by a number that's a little bigger than 3.
- You can make reasonable estimates for the **diameter** by dividing the circumference by 3, but you can make even closer estimates by dividing by a number that's a little bigger than 3.

So, what IS that number?

If you want a closer estimate, the number to use is about 3.14. It's impossible to say what it is exactly because the decimals go on forever in the number that you multiply the diameter by to get the circumference. In 1706, this important number was given the name "pi", which is the Greek letter " π ." π is the first letter of the Greek word "perimetros" which refers to the distance around the circle in this context.

π

Want to see more decimal places of π ? Look for a button with that symbol on a scientific calculator. If you have a smartphone, it probably has a scientific calculator built in.

- If you have an Android phone, you may be able to access the scientific calculator by swiping from right to left in your calculator app. If that doesn't work, try following the directions here: <u>https://www.wikihow.com/Use-an-Android-Calculator</u>
- If you have an iPhone, you can access the scientific calculator by opening the calculator app and turning the phone sideways. You can see an example of what this looks like here: https://support.apple.com/guide/iphone/calculator-iph1ac0b5cc/ios (If that doesn't work, you may have the rotation lock turned on. Here's how to fix that: https://support.apple.com/guide/iphone/calculator-iph1ac0b5cc/ios (If that doesn't work, you may have the rotation lock turned on. Here's how to fix that: https://support.apple.com/en-us/HT204547. Note that the instructions are different for phones with and without home buttons.)

If you don't have a smartphone or your phone doesn't have a calculator, you can also find scientific calculators online. Here's a good one: <u>https://www.desmos.com/scientific</u>



Find the π button on your calculator and see how many digits it shows you.

Write them here: ______

Want to see even more digits?

You can see the first *million* digits of π here: <u>https://www.piday.org/million/</u>. And π keeps going even beyond that... but you don't need to learn all those numbers! Luckily, for most real-world circle problems, using 3.14 is close enough, and when you're estimating, just thinking of it as a little more than 3 works out just fine.

Use this answer key to check your answers to the activity questions. Note that you may not have used the same words in your answers. Look at whether the ideas in these answers match your ideas. You may not get exactly the same numbers in your answers. Most of these activities involve estimating. Look at whether your numbers are close to the numbers in the answers.

p. 2 How is the diameter related to the circumference?

The circumference is a little bit more than 3 times the diameter. It takes a little more than 3 diameters end on end to measure the circumference.

p. 2 Where do you see this relationship in the table?

If you triple the diameter it is about the size of the circumference or if you divide the circumference by 3 you get close to the value of the diameter

p. 2 How could you verify with your cut string lengths?

If you take your diameter and cut 2 more pieces of string the length of the diameter and place them end on end, it should almost equal the length of the circumference string. You can also use the diameter string as the measuring tool, it should take 3 of them to almost make the circumference.

p. 2 Based on what you have observed, if you knew the length of the diameter, how could you figure out the approximate circumference? If you knew the length of the circumference, how could you roughly determine the diameter?

You could triple the length of the diameter (multiply it by 3) to find the circumference.

You could divide the circumference by 3 or you could take 1/3 of it to find the diameter.

p. 3 What do you notice? How does this relate to what you discovered about the diameter and circumference from the table and with string lengths?

Here is an example graph. Your points will be different because you measured different objects, but they should roughly form a straight line that is close to the line formed by the points in the graph below.



Here are some things we noticed. You might have noticed different things!

- The points seem to line up.
- If the points are connected, it forms a line.
- For every movement across the x axis, you go up 3 times as much on the y axis.

Here are two ways you might connect what you see in the graph to the measurements you made:

• Because the points all line up, it suggests that if you measured more circles, the points you would get from those would also line up.



• The relationship between the circumference and the diameter of the circle can be seen in the graph because the line goes up about 3 for every 1 it goes over. In other words, the slope of the graph is about 3. (If you haven't heard of slope, it's a number that describes how steep a line is.)

p. 4 Based on your graph, if the circumference was 7 inches, about what would the diameter be?

Look at the imaginary line that your points form. You can draw the line in if you want. If the circumference is 7 inches, then the diameter would have to be a little more than 2 inches to make the point be on the line.

p. 4 Based on your graph, if the diameter was 4 inches, about what would the circumference be?

If the diameter was 4 inches, then the circumference would have to be around 12 inches to make the point be on the line.

p. 4 Is there a way to calculate your answers without using the graph or table? Explain.

Since it appears that 3 times the diameter is close to the circumference you could multiply by 3 to arrive at a circumference that is close to the original circumference

p. 4 What would remain the same and what would change in your table if you measured your diameter and circumference in centimeters? What would remain the same or change in your graph? Explain.

In the table, the relationship of 3 and a little bit more diameters equaling the circumference would remain the same. Just the values in the table might appear different. For example, if they originally had a diameter of 6 inches, in centimeters that would be about 15 cm and the circumference would change from around 18 inches to around 45 cm.

In the graph, the plotting of the points would still form a line but what would change are the increments on the graph. Since centimeters are smaller than inches the increments for the diameter would probably need to be in increments of 5 and the increments for the circumference may need to adjust perhaps to increments of 10.

p. 5 Were there any measurements that you thought were unreasonable? If so, what was it about them that made you think that?

The measurements that don't seem to fit are the ones for Pattie's pie pan and the medium lid. For all the other measurements, the circumference is about three times as big as the diameter, but for Pattie's pie pan, it's only about twice as big and for the medium lid, it's more than five times as big!

Add the data above to the graph that you made in the last activity. Use a different color or a different kind of mark (like an x or a star instead of a dot) so you can tell the added data from your original data.

Your graph with the new data might look like this. Note that the locations of the new points are approximate.



p. 5 How does the graph affect your ideas about whether the measurements were reasonable?

You might notice on the graph that two points are not in line with the rest of the points. Those two points represent Pattie's pie pan and the medium lid. The points that are not in line have unreasonable measurements. The rest of the points have reasonable measurements.

p. 6 How can you tell if they are more reasonable now?

There are two ways you might fix the unreasonable measurements depending on whether you think it is the circumference or the diameter that is off.

Fixing the circumference might give you something like this:

Object	Diameter	Circumference
Patti's pie pan	9″	18 1⁄2″
Medium lid	5 ¾″	27 ½″

Fixing the diameter might give you something like this:

Object	Diameter	Circumference
Patti's pie pan	6″	18 1⁄2″
Medium lid	9″	27 1⁄2″

Note that these "fixed" numbers are guesses at what the correct measurements might have been. These are more reasonable measurements because they fit the relationship of the circumference being a little more than three times the diameter. When you add them to your graph, you should see that they line up with the other points much better than they did before.

p. 6 Estimate the diameter of the London Eye.

The diameter is about 120 meters because 120 multiplied by 3 is almost 375.

p.7 How high above the water do you think the people all the way at the top are?

I think they are **123** meters above the water because **it looks like the bottom of the ferris wheel is about 3 meters above the water**.

p.7 How tall do you think the buildings in the background are?

I think building A is about **25** meters tall because **it looks less than a fourth as tall as the ferris wheel.**

I think building B is about 60 meters tall because it looks about half as tall as the ferris wheel.

I think building C is about **30** meters tall because **it looks about a fourth as tall as the ferris wheel.**

p. 7 About how long do you think the spokes that run from the center to the outside are?

I think they are 60 meters because they go halfway across the ferris wheel.

p.7 What math word describes the spokes?

Radius

p. 8 Use a piece of string to measure the circumference of your head.

Answers will vary, but most adults' heads are between 20 and 25 inches around.

p. 8 Based on the circumference of your head, estimate how many inches across a hat would have to be to fit you comfortably. (It's okay that your head isn't perfectly circular, you're just estimating!) How did you make your estimate?

Answers will vary. To estimate your hat size, divide the circumference of your head by 3. For a closer estimate, go with a number that's a little smaller than what you get when you divide by 3.

p. 8 How did you make your estimate?

To estimate your hat size, divide the circumference of your head by 3. For a closer estimate, go with a number that's a little smaller than what you get when you divide by 3.

p. 8 Here are some measurements three people took and the hat sizes they bought. For each person, decide whether the hat would be too big, too small, or just right and write that in the table.

Name	Head Circumference	Hat Size	How does it fit?
Usha	23″	7 1⁄2	just right
Charlie	24 1⁄2″	6	too small
Liza	21″	7 7/8	too big

p. 8 How did you decide whether the hats would fit?

You might have divided each circumference by 3 and compared that to the hat size.

p. 9 If the diameter of each of the tires on bicycle A is 16", what do you estimate the diameter of the tires on bicycle B is? How did you make your estimate?

It looks like the diameter of the tires on bicycle B is about two times the diameter of the tires on bicycle A, so their diameter is probably about 32".

p. 9 About how many inches do you think each bicycle travels when the tires make one complete rotation?

I think bicycle A travels about 50 inches because that is a little more than three times the diameter. I think the whole circumference of the wheel will touch the ground in one complete rotation.

I think bicycle B travels about 100 inches because 100 is a little more than three times 32.

p. 10 About how long would be a good string length for you?

That's your personal choice! You might measure the distance from the ground to your waist to decide.

p. 10 About how big a space would you need to practice this trick safely so you don't hit anything (or anyone!) with the yo-yo? How did you decide what would be safe?

You would want a space that is big enough that the yo-yo could go all the way out to the end of the string without hitting anything. One way to be sure you are safe is to imagine a circle with the yo-yo string as the radius and find a place to practice where nothing is inside that circle.

p. 10 About how far would the yo-yo travel if you performed the trick once?

You need to find the circumference of the circle the yo-yo would make when you swing it around. Start by doubling the radius to find the diameter and then multiply by three and go a little higher than that number to find the circumference. For example, if your yo-yo string is 3 feet long, then the diameter of the circle would be 6 feet and the circumference would be a little more than 18 feet.

p. 10 In the video, Paul Yath, a yo-yo champion, made his yo-yo go around 11 times! About how far do you think his yo-yo traveled during that trick?

You have to guess how long his yo-yo string is. If it were 3 feet long, then the yo-yo would travel a little more than 18 feet in one revolution (as described in the previous answer) and in 11 revolutions, it would go 11 times as far... that's about 200 feet!

p. 12 Find the π button on your calculator and see how many digits it shows you.

Here are some more digits of π: **3.14159265358979323846264338327950**...

(Your calculator may round off several digits in its display.)



A tree has a 2-foot diameter. About how long would a string be if you wrapped it once around the tree?

- A) 8 inches
- B) 2 feet
- C) 4 feet
- D) 6 feet
- E) 8 feet

Question 2

About how much longer is the circumference of a 2½ foot diameter tree than the circumference of a 2-foot diameter tree?

- A) About ½ foot
- B) About 1¹/₂ feet
- C) About 3 feet
- D) About 5 feet
- E) About 7¹/₂ feet

Question 3

The first Ferris Wheel, named after its designer, George Ferris, was built in 1893. The diameter of the first wheel was 76 m. About what distance would a rider travel one time around?

- A) 76 m
- B) 152 m
- C) 228 m
- D) 304 m
- E) 380 m





A circle is inscribed in a square. About how many feet longer is the perimeter of the square than the circumference of the circle?

- A) 2 feet
- B) 4 feet
- C) 6 feet
- D) 12 feet
- E) 16 feet



Question 5

The minute hand on a clock is 3 inches long. About what distance does the tip of the minute hand travel in 1 hour?

- A) 6 inches
- B) 12 inches
- C) 18 inches
- D) 24 inches
- E) 30 inches



Question 6

A circular trampoline has a diameter of 48 inches. Protective padding that encircles the trampoline comes in strips that are 5 inches wide. About how many feet of padding would have to be purchased to have enough to completely go around the trampoline?

- A) 12 feet
- B) 13 feet
- C) 18 feet
- D) 24 feet
- E) 48 feet





A standard running track consists of two parallel straightaways and 2 bends which are semicircles. What is the approximate distance around the running track?



Question 8

In 2018, an unusual spinning ice disk formed in the Presumpscot River in Westbrook, Maine. From drone photography, the disk was estimated to be about 300 feet across. Birds would sit on the edge of the ice and travel in a counter clockwise motion as the disk spun against the river current. About how far would a bird travel halfway around the revolution of the disk?

- A) 150 feet
- B) 250 feet
- C) 350 feet
- D) 450 feet
- E) 900 feet





If each grid represents one unit of measure, about how many units is the circumference of the circle graphed?

- A) 10 units
- B) 20 units
- C) 30 units
- D) 40 units
- E) 50 units



Question 10

Two circles share the same center. If the diameter of the larger circle is 15 centimeters, what is the circumference of the smaller inner circle?

- A) 3 π
- B) 5 π
- C) 7 π
- D) 9π
- E) 12 π



Circle Packet Test Questions

Answer Key for Test Questions

Question 1	D
Question 2	В
Question 3	С
Question 4	В
Question 5	С
Question 6	В
Question 7	D
Question 8	D
Question 9	С
Question 10	D