How Grand is Your Total?
A unit teaching properties, equality, and the power of place value

College and Career Readiness Standards for Mathematics:

- Levels: A – C
- Domains: Operations and Algebraic Thinking, Numbers and Operations in Base Ten, and Number and Operations – Fractions
- Standards for Mathematical Practice:
  - Make sense of problems and persevere in solving them (MP.1)
  - Attend to precision. (MP.6)
  - Look for and make use of structure. (MP.7)
- Focus: *Focusing strongly where the standards focus.* "…selection of priority content addressing a clear understanding of place value and its connection to operations… leads to a deeper understanding of the properties of operations at subsequent levels…"

Unit Length: 11 lessons, 2 – 4 hours each

See **Teacher Resources** document for support to this unit plan https://www.dropbox.com/sh/1ubpubqo8ehstg7/AADRgoliZ7VVRla3R5rdF8J9a?dl=0.

Introduction

Welcome to *How Grand is Your Total*! This unit uses puzzles and questions with multiple solution paths to engage students in learning about properties of numbers and operations, the meaning of equality, and the power of place value. You may have some questions about whether this unit will work for your students. Here are answers to some questions you may have.

Why do the topics of properties, equality, and place value deserve a whole unit?

While these topics may take up only a few pages in a standard textbook, engaging deeply with these concepts will build students' current and future mathematical and reasoning skills in significant ways. In this unit, students will explore operations deeply. In the process, they will make sense of procedures for addition, subtraction, multiplication, and division and even make connections to fractions. Through this deep exploration, students will develop a comfort level and confidence that will help them apply procedures fluently.

This unit is also building foundational concepts for algebra. Specifically it lays the groundwork for formal learning about properties, inverse operations, and solving equations. In fact, even students working at the lowest levels will be reasoning algebraically from the very first lesson.

How is this unit different from traditional instruction on these topics?

The primary learning activities in this unit are puzzles and "open middle" problems that engage students in deep reasoning. Open middle problems (from https://www.openmiddle.com/) are problems where students begin with both the question and the answer and work to figure out the middle part of the problem. For example, instead of being asked to practice adding three two-digit numbers, an open middle problem might challenge students to come up with three, two-digit numbers that add to 100. Reasoning about the open middle problem still addresses the skills involved in adding, but it also requires students think more deeply as they wrestle with the puzzle. This exposes students to tasks with a greater depth of knowledge." In this unit, students will learn to value and nurture their own ways of thinking, to give themselves time to struggle, and to communicate mathematically.

Will my students be willing to do puzzles and open middle problems that aren't connected to a real world context?

While it is true that connecting math real world contexts is an effective way of engaging adult learners in math, it is not the only way. We have seen that adult learners are eager to take on interesting and challenging puzzles and that they develop confidence and persistence through doing so. The puzzles and tasks in this unit are carefully organized and scaffolded to keep students in a place of productive struggle. In this unit, students will experience success at the first Standard for Mathematical Practice (from the College and Career Readiness Standards for Adult Education) – Make sense of problems and persevere in solving them. This will prepare them to go confidently in any mathematical direction in their future learning.

How does this unit promote diversity, equity, and inclusion in my classroom?

Okay, maybe you weren't asking yourself this, but you should know that opening up your math class so that students can be creative, express themselves in different ways, and learn to value their own reasoning creates a more equitable environment where students of diverse backgrounds and cultures can thrive. Using these lesson activities and this style of instruction is one of many important steps you can take to make your classroom more equitable and inclusive for all learners.

Lesson 1: Properties of Numbers

Lesson Objectives:

- *observe 'always true' properties [Focus]*
- *begin to develop habits of problem solving [MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by … looking for entry points to its solution. They analyze givens, constraints, relationships, and goals.]*
- *prepare to use properties of operations to add and subtract [CCRS Levels A & B] in upcoming problems*

True or False? (1.OA.7)

Purpose: for students begin to identify things that are always true about operations (properties).

2 + 5 = 7
2 + 5 = 5 + 2
7 = 7
7 + 0 = 0 + 7
7 = 6 + 1

1) One at a time, ask students to identify number statements such as these as true or false. Use the wording "is the same amount as" for the equal sign to help students build their understanding of equality.
2) Ask questions such as:
   - Would the number sentence still be true if it were subtraction instead of addition?
   - Would the number sentence still be true if we changed the order of __?
   - Would the number sentence still be true if we replaced __ with a larger number, such as 100?

   to draw out a student-created list of 'always true' statements such as:

   - adding zero to anything yields what you started with
   - adding can be done in any order
   - subtraction cannot be done in any order
   - the 'math' (addends) can be on either side of the equal sign
   - a number is equal to itself

Open Number Sentences (1.OA.7)

Purpose: to introduce a more open, algebraic set of number sentences to further the conversation about properties.

1) Using a selection of open number sentences such as these, ask students:
   - What will complete the sentence?
   - How do you know what number will work in the box to make a true statement?

2) Connect student responses to their list of things that are always true about operations. For example, connect 4 + 5 = 5 + ___ to “adding can be done in any order.” If appropriate, mention that finding an unknown amount is algebra.

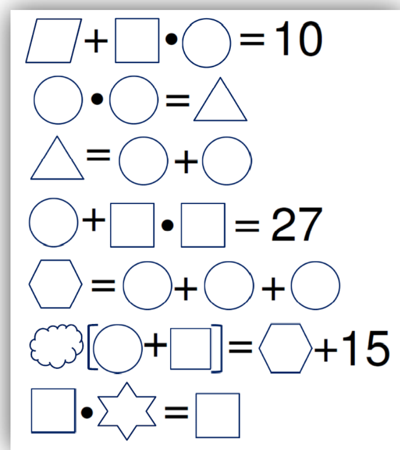
3) Tell students they will be working on puzzles during this unit that will help them understand some big ideas about how math works. The puzzles will look simple, but in fact will require them to think deeply.

4 + 5 = □	**or**	**4 + □ = 9**
4 + 5 = □ + 4	**or**	**4 + 5 = 5 + □**
9 = □		
9 + □ = 9	**or**	**□ + 0 = 9**
9 = 6 + □	**or**	**9 = □ + 3**

Properties of Numbers Challenge (MP.1)

Purpose: to challenge students to simultaneously think about several properties in order to solve a puzzle.

[**Note:** If your students need more successes in order to maintain persistence, move this activity and handout to after Lesson 2: Addition.]



1) Tell students that these “things which are always true about computation” are called properties. Students will use what they know about properties to figure out what belongs in the empty spaces of this puzzle. They may notice, for example:

   - There are two equations with circles and a triangle. Therefore, the circle represents a number that when added to itself or multiplied by itself, yields the same answer.

   - The hexagon must represent a multiple of 3 because it is the result of three of the same number being added together.

2) Before asking students to work independently, check for understanding of the notation “•” and “**[ ]**” instead of “**x**” for multiplication.

3) Consider several intermediate, whole-group debriefs in response to what you see developing with your students’ work. Questions may include:

   - What seems to be true about [shape]?

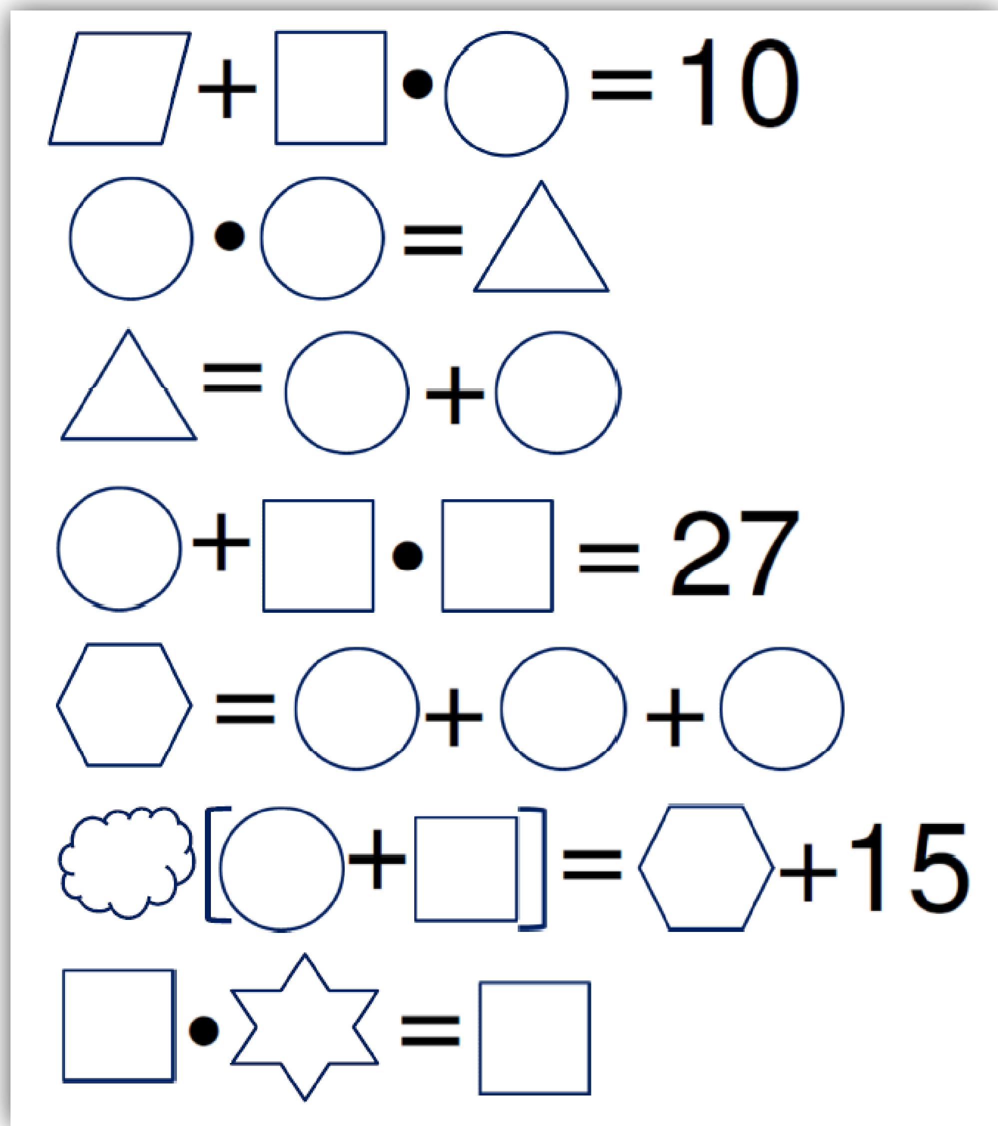
   - Where are you starting in the puzzle? Why?

   - What are all the possible answers for [a particular row] if you don’t consider other rows first?

Name: ______________________

Properties of Numbers Challenge

Using what you know about the properties of numbers, look for patterns and determine the value of each shape. Each shape represents a different single-digit, whole number.



Lesson 2: Addition

Lesson Objectives:

- *understand place value [2.NBT.1]*
- *use place value and properties of operations to add and subtract [CCRS Levels A & B]*

Sums to 100 (2.NBT.6)

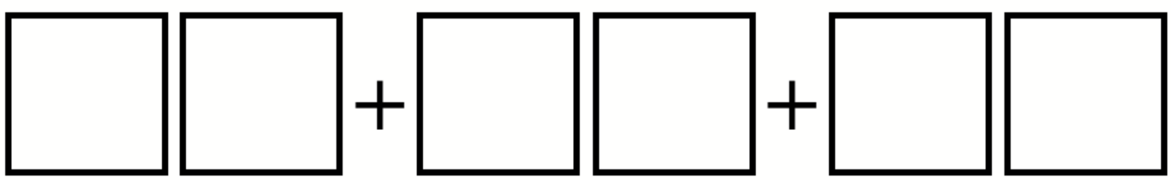
Purposes:

- *to introduce students to Open Middle puzzles.*
- *to build place value understanding and use it as a strategy to add.*

1) Display the Sums to 100 puzzle.

SUMS TO 100

Directions: Using the digits 1 to 9 at most one time each, fill in the boxes to create the closest possible sum to 100.



2) Offer the Geogebra tool option found at https://www.geogebra.org/m/zq8RFfJX or suggest students use scraps of paper, each with a digit on it for easy rearranging.
3) Tell students that this is one of the puzzles that will allow them to think deeply about math ideas while they work toward solutions. Set expectations about talking about thinking rather than their answers so everyone has time to think.
4) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

   Questions to move students along:

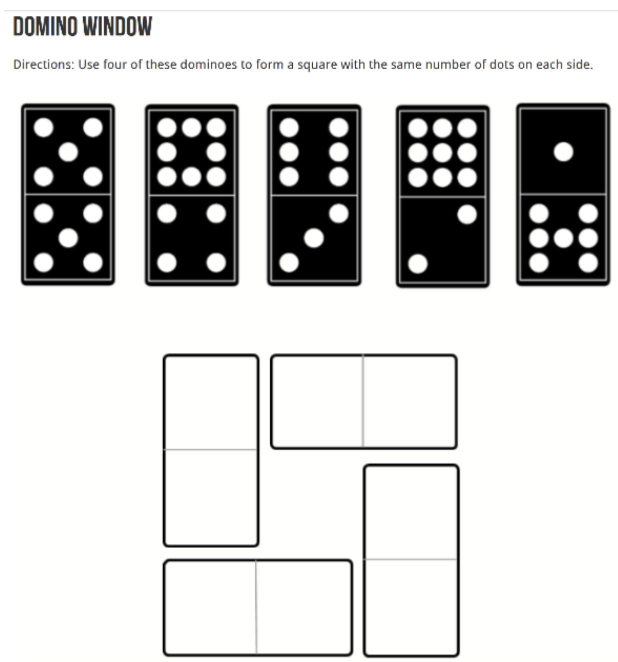
   - If you could repeat digits, could you make an arrangement that adds to 100? (then a student can adjust from there)
   - Follow up: How can we use this to fill in the boxes using each digit only once?
5) After all students have at least one sum to 100, compile the answers where everyone can see.
   - What patterns are you starting to notice about correct answers? (2.NBT.9) Student examples:
     - Ones must add to 20
     - Use 4, 3, and 1 in the tens place
     - Use 1, 2, and 5 in the tens place
     - Tens must add to 8
     - Must be one even number and 2 odd numbers
6) Allow time for students to find as many sums to 100 as they can.

Multiple sums to 100:

19 + 28 + 53	19 + 27 + 54	19 + 36 + 45	18 + 37 + 45
19 + 23 + 58	19 + 24 + 57	19 + 35 + 46	18 + 35 + 47
18 + 23 + 59	17 + 24 + 59	16 + 35 + 49	15 + 37 + 48
18 + 29 + 53	17 + 29 + 54	16 + 39 + 45	15 + 38 + 47
13 + 29 + 58	14 + 29 + 57	15 + 39 + 46	17 + 38 + 45
13 + 28 + 59	14 + 27 + 59	15 + 36 + 49	17 + 35 + 48

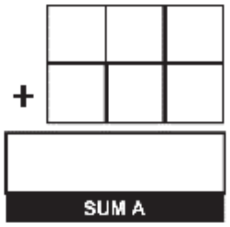
OPTIONAL HOMEWORK:

- Domino Window (1.OA.2)



Two answers:

- How Grand is Your Total – Sum A (Directions: Using the digits 0 – 6, make the largest possible sum.) (2.NBT.7)



Solutions:

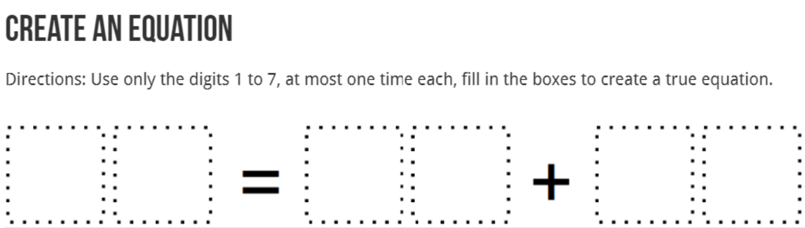
642 + 531 = 1173

532 + 641 = 1173

541 + 632 = 1173

542 + 631 = 1173

- Create an Equation (2.NBT.5 & 2.NBT.9)



Unique solutions:

43 = 16 + 27
43 = 17 + 26
47 = 32 + 15
47 = 35 + 12
57 = 21 + 36
57 = 31 + 26
61 = 34 + 27
61 = 37 + 24
62 = 15 + 47
62 = 45 + 17
67 = 14 + 53
67 = 54 + 13
71 = 25 + 46
71 = 26 + 45
74 = 23 + 51
74 = 53 + 21
75 = 12 + 63
75 = 13 + 62
76 = 41 + 35
76 = 45 + 31

Lesson 3: More Addition

Lesson Objectives:

- *understand place value [2.NBT.1]*
- *use place value and properties of operations to add and subtract [CCRS Levels A & B]*

Close to 1000 (2.NBT.7 & 3.NBT.2)

Purpose: to deepen and expand place value understanding to three-digit numbers and use it as a strategy to add.

1) Display the Close to 1000 puzzle and offer the Geogebra tool option found at https://www.geogebra.org/m/mYTjP7Fc.

CLOSE TO 1000

Directions: Using the digits 1 to 9 exactly one time each, place a digit in each box to make the sum as close to 1000 as possible.

2) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

   Questions to move students along:

   - How do you know you can't get any closer to 1000?
   - What should be true about the hundreds places of your three numbers? (2.NBT.9)
   - How do the tens places affect your answer? (2.NBT.9)
   - Is exactly 1000 even possible?

3) Collect students' answers and ask for patterns. Encourage using patterns to find more answers:

   139 + 286 + 574 = 999

   189 + 243 + 567 = 999

   189 + 247 + 563 = 999

   189 + 274 + 536 = 999

   196 + 328 + 475 = 999

   198 + 326 + 475 = 999

4) If students assumed answers must be close to but not over 1,000, ask a follow up question.

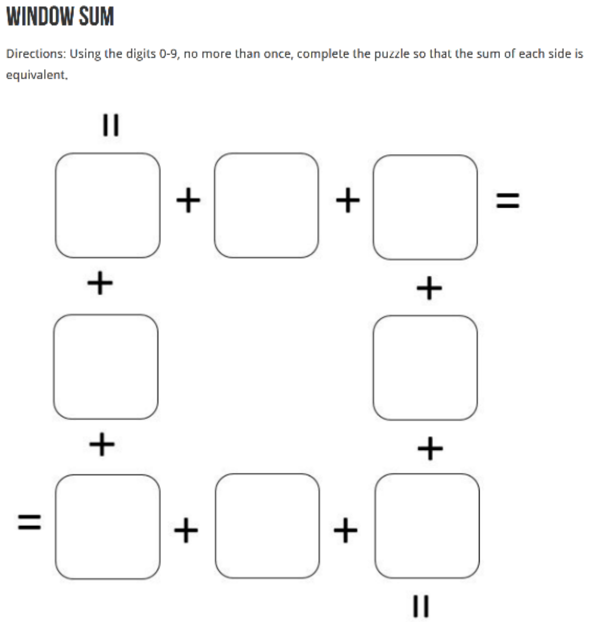
   - What is the closest you can get but be *above* 1,000?

   Example solutions that generate (shuffle the numbers in each place value) other solutions:

589	598	598	497	489
276	267	276	386	367
+143	+143	+134	+125	+152
1008	1008	1008	1008	1008

OPTIONAL HOMEWORK:

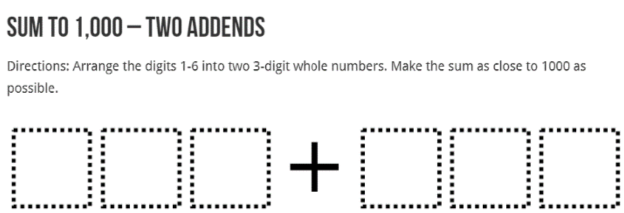
- Window Sum (1.OA.3) (multiple answers)



One solution:

12
4 8 0 12
6 5
12 2 3 7
12

- Sum to 1,000 – Two Addends (2.NBT.7)



Some of the solutions:

431 + 562 = 993

432 + 561 = 993

Lesson 4: Equality

Lesson Objectives:

- *understand the meaning of the equal sign [1.OA.7]*
- *introduce the concept of an equation, a variable, and the meaning of the equal sign, all within the context of addition and subtraction [Focus of CCRS Level A]*
- *move from computational thinking to relational thinking [bridge from arithmetic to algebra]*
- *use the equal sign consistently and appropriately [MP.6]*

Considerations during teaching equality:

Move student thinking toward the Relational Thinking needed for algebra so students:

- Consider the number sentence as a whole.
- Analyze the structure.
- Generate productive solutions.

Use the language *is the same as* instead of *equal*. The equal sign does not mean *put the answer here*.

THINKING ABOUT STUDENT THINKING

Computational Thinking	**Relational Thinking**
Operational and arithmetic focus. Carry out a step-by-step sequence.	Identify *relationships* among and within expressions and equations. Examine expressions and equations in their entirety. Flexible thinking with a basis in the foundation of *properties of number operations*.

True or False? (2.NBT.9)

Purpose: to support student understanding of place value using relational thinking.

56 = 50 + 6	94 = 70 + 24	63 – 28 = 60 – 20 – 3 – 8
87 = 7 + 80	86 = 6 + ☐	63 – 28 = 60 – 20 + 3 – 8
93 = 9 + 30	47 + 38 = 40 + 30 + 7 + 8	
94 = 80 + 14	24 + 78 = 78 + 20 + 2 + 2	

1) Use a selection of number sentences such as these.
2) In pairs and one number sentence at a time, ask students to mentally consider whether these statements are true or false and why. Listen for answers that include reasoning about place value and thinking about how both sides of the equal sign relate:
   - Looking at 94 = 80 + 14, I see moving the 10 from the 14 to make it 90 + 4, which is 94. This is true.
   - Looking at 47 + 38 = 40 + 30 + 7 + 8, on the right I see 40 + 7 (like the 47 on the left) as well as 30 + 8 (like the 38 on the left). They are all added, so this is true.
   - Looking at 63 – 28 = 60 – 20 – 3 – 8, on the right I see the – 20 and – 8 making up the – 28 like on the left, but on the right that should be + 3 to make up 60 + 3 like the 63 on the left. So, this is false.

Make it Equal (1.OA.7)

Purpose:

- *to formatively assess students' understanding of the meaning of the equal sign.*
- *to move student's thinking to relational thinking and an understanding of equality.*

1) Display the Make It Equal puzzle.

MAKE IT EQUAL

Directions: Using the digits 1 to 9 at most one time each, place a digit in each box to create a true statement.

□=□+□=□+□+□

2) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

   Questions to move students along:

   - Are there numbers that are helpful to use first or save for last?
   - What numbers can*not* go in the first box?
   - What would a drawing/model look like to match this equation?

3) Display an interesting wrong answer, such as 9 = 8 + 1 = 9 + 4 + 5, without identifying the author.

   - Ask students to talk about what's right. (they are thinking about 9s with 8 + 1 and 4 + 5)
   - Ask students to talk about what needs to be adjusted. (the value of 9 + 4 + 5 needs to be equivalent to both the value of 8 + 1 and 9)

4) After all students have at least one correct solution, begin to compile answers where everyone can see. Ask about patterns.

5) Allow additional time for students to find as many equations as they can.

   Forty-eight unique solutions (allowing for permutations):

   8 = 2+6 = 1+3+4
   8 = 2+6 = 1+4+3
   8 = 2+6 = 3+1+4
   8 = 2+6 = 3+4+1
   8 = 2+6 = 4+1+3
   8 = 2+6 = 4+3+1
   8 = 6+2 = 1+3+4
   8 = 6+2 = 1+4+3
   8 = 6+2 = 3+1+4
   8 = 6+2 = 3+4+1
   8 = 6+2 = 4+1+3
   8 = 6+2 = 4+3+1
   9 = 1+8 = 2+3+4
   9 = 1+8 = 2+4+3
   9 = 1+8 = 3+2+4
   9 = 1+8 = 3+4+2
   9 = 1+8 = 4+2+3
   9 = 1+8 = 4+3+2
   9 = 2+7 = 1+3+5
   9 = 2+7 = 1+5+3
   9 = 2+7 = 3+1+5
   9 = 2+7 = 3+5+1
   9 = 2+7 = 5+1+3
   9 = 2+7 = 5+3+1
   9 = 4+5 = 1+2+6
   9 = 4+5 = 1+6+2
   9 = 4+5 = 2+1+6

9 = 4+5 = 2+6+1	9 = 5+4 = 6+1+2	9 = 7+2 = 5+3+1
9 = 4+5 = 6+1+2	9 = 5+4 = 6+2+1	9 = 8+1 = 2+3+4
9 = 4+5 = 6+2+1	9 = 7+2 = 1+3+5	9 = 8+1 = 2+4+3
9 = 5+4 = 1+2+6	9 = 7+2 = 1+5+3	9 = 8+1 = 3+2+4
9 = 5+4 = 1+6+2	9 = 7+2 = 3+1+5	9 = 8+1 = 3+4+2
9 = 5+4 = 2+1+6	9 = 7+2 = 3+5+1	9 = 8+1 = 4+2+3
9 = 5+4 = 2+6+1	9 = 7+2 = 5+1+3	9 = 8+1 = 4+3+2

Solve Me Mobiles

Purpose: to build algebraic and relational thinking.

1) Share access to Solve Me Mobiles https://solveme.edc.org/Mobiles.html. If students create a free account, they can track their puzzle progress.
2) Do a few puzzles as a class and give time for students to explore.
3) After use in class, strongly suggest for homework on multiple days.

OPTIONAL HOMEWORK:

- Solve Me Mobile puzzles: https://solveme.edc.org/Mobiles.html
- Equality

EQUALITY

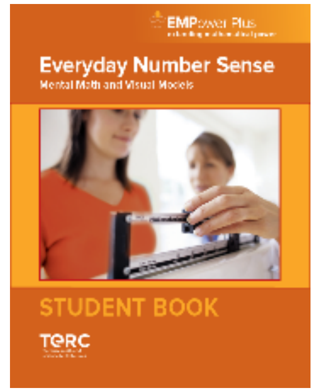
Directions: Using the digits 0-9 at most one time each, fill in the boxes below to make a true statement.

□ + □ = □ + □ = □ + □ = □ + □ = □ + □

Solutions should add to nine with some arrangement of:

0 + 9 = 1 + 8 = 2 + 7 = 3 + 6 = 4 + 5

- EMPower's *Everyday Number Sense: Mental Math and Visual Models* student book (https://www.terc.edu/empower):
  - Make it True (p26 and 72)
  - Check Both Sides of the Equal Sign (p. 27-28 and p. 73-74)
  - Practice Check Both Sides of the Equal Sign (addition p. 89 and subtraction p. 90)

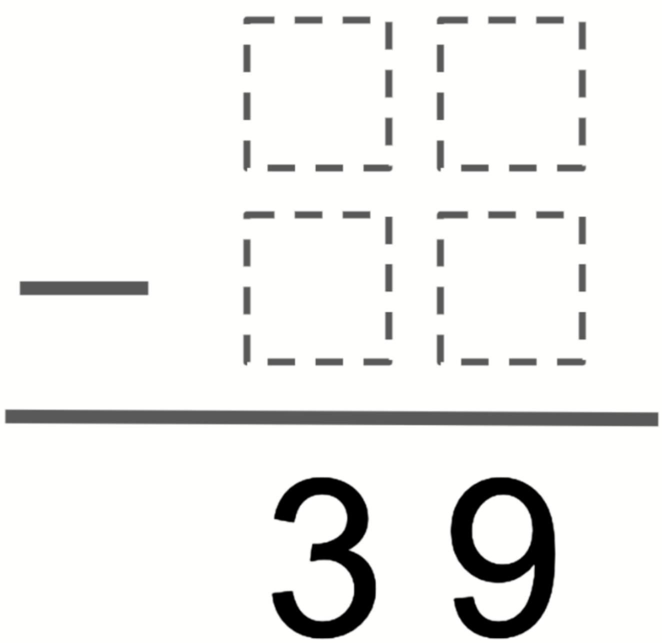


PREWORK:

- Subtraction with Regrouping 2

SUBTRACTION WITH REGROUPING 2

Directions: Using the digits 1 to 9 at most one time each, fill in the boxes to make the difference equal to 39.



Solutions:

52-13, 53-14, 56-17, 58-19, 63-24, 64-25, 67-28, 68-29, 71-32, 74-35, 75-36, 78-39, 81-42, 82-43, 85-46, 86-47, 87-48, 91-52, 92-53, 93-54, 96-57, 97-58, 98-59

Lesson 5: Subtraction

Lesson Objectives:

- *understand place value [2.NBT.1]*
- *use place value and properties of operations to perform multi-digit arithmentic [CCRS Level B]*
- *construct viable arguments and critique the reasoning of others [MP.3]*
- *understand the meaning of the equal sign [1.OA.7]*
- *use the equal sign consistently and appropriately [MP.6]*

Subtraction to Get the Smallest Difference (2.NBT.7 & 3.NBT.2)

Purpose: to use place value understanding as a strategy to subtract and predict the value of results.

1) Display the Subtraction to Get the Smallest Difference puzzle.

SUBTRACTION TO GET THE SMALLEST DIFFERENCE

Directions: Using the digits 1 to 9, at most one time each, fill the boxes below to create the smallest possible difference.

□□□ − □□□ = ?

2) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

   Questions to move students along:

   - Is it possible to get a 2-digit number as your answer?
   - What was different about the problem in the prework, □□ - □□ = 39?

3) Collect student's answers and ask for their strategies. Encourage using those strategies to find more answers.

   Multiple solutions:

   412 – 398 = 14

   512 – 498 = 14

   612 – 598 = 14

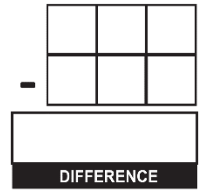
   712 – 698 = 14

4) ***Follow up:*** Use the same problem to get the largest difference.

OPTIONAL HOMEWORK:

- How Grand is Your Total? Difference (Directions: Using the digits 0 – 6, make the largest possible difference.)

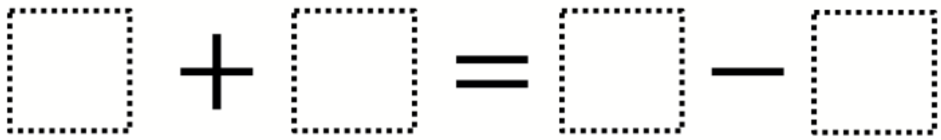
  Solution: 654 – 012



- Adding and Subtracting within 10 (K.OA.2)

ADDING AND SUBTRACTING WITHIN 10

Directions: Using the digits 1 to 9 at most one time each, place a digit in each box to make a true statement.



Solutions:

1+2 = 7-4

3+1 = 6-2

1+3 = 6-2

1+4 = 7-2

5+1 = 8-2

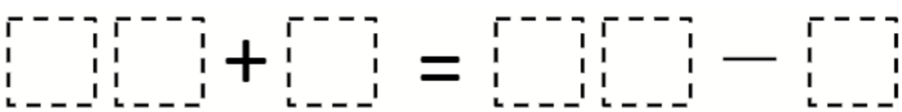
1+6 = 9-2

3+5 = 9-1

- Add Some, Subtract Some (1.NBT.4)

ADD SOME, SUBTRACT SOME

Directions: Use the digits 1-9, at most once, to complete the equation.
What is the greatest result you can make?
What is the least result you can make?



Some solutions:

27 – 8 = 14 + 5

25 – 8 = 13 + 4 (or 14 + 3) ~ least result of 17 = 17

95 – 2 = 87 + 6 (or 86 + 7) ~ greatest result of 93 = 93

- Adding and Subtracting Two-Digit Whole Numbers (2.NBT.5 & 2.NBT.7)

ADDING AND SUBTRACTING TWO-DIGIT WHOLE NUMBERS

Directions: Directions: Use the digits 0 to 9, at most one time each, to make a true statement.

□□ – □□ = □□ + □□

Questions to move students along or debrief:

- What's your best first move? Does it make more sense to start with the addition or subtraction?
- What's the greatest value you can make both sides equivalent to? Smallest value?

  85 – 40 = 26 + 19 (both sides equal 45)

  78 – 19 = 24 + 35 (both sides equal 59)

  97 – 28 = 13 + 56 (both sides equal 69)

  98 – 15 = 47 + 36 (both sides equal 83)

Lesson 6: Multiplication

Lesson Objectives:

- *use place value and properties of operations to perform multi-digit arithmetic [Level B]*
- *begin to learn partial products [2.NBT.3]*
- *identify arithmetic patterns and explain them using properties of operations [3.OA.9]*
- *look for and make use of structure [MP.7]*
- *interpret products of whole numbers as the total number of objects in groups [3.OA.1]*
- *understand properties of multiplication [Level B]*

True or False? (3.OA.1)

Purpose:

- *to support connections between addition and the meaning of multiplication.*
- *to support relational thinking by learning to interpret multiplication as the total number of objects in groups or sets of objects.*

1) Use a selection of number sentences such as these. Model thinking rather than reading the symbols. (*i.e.,* "3 sets of 7 is the same as adding 7 three times. This is true.")
2) Ask student pairs to take turns reading the relationship out loud and then determine whether the statement is true or false.

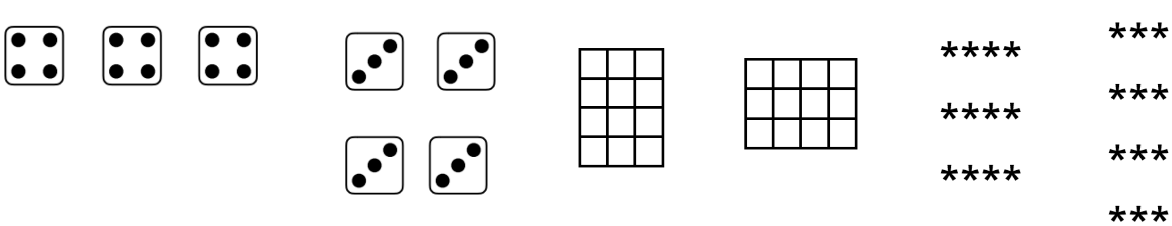
3 x 7 = 7 + 7 + 7	6 x 7 = 5 x 7 + 7
3 x 7 = 14 + 7	8 x 6 = 8 x 5 + 6
4 x 6 = 12 + 12	7 x 6 = 7 x 5 + 7
3 x 8 = 2 x 8 + 8	9 x 7 = 10 x 7 – 7

Drawing Multiplication (3.OA.2)

Purpose:

- *to support understanding what multiplication means.*
- *to set the foundation for interpreting division in a future lesson.*

1) Ask students to represent 3 x 4 using a drawing, or manipulatives such as tiles, pennies, scraps of paper, etc.
2) Use language of sets, groups, rows, columns, etc. to describe student representations of three 4s and four 3s.



3) Save these representations for the Division lesson.

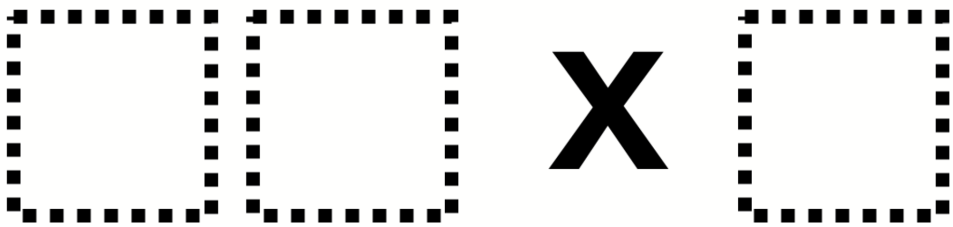
Multiplying a Two-Digit Number by a Single-Digit Number (3.NBT.3)

Purpose: to extend understanding of place value from the value of digits in a single number to understanding the power of place value within multiplication.

1) Display the Multiplying a Two-Digit Number by a Single-Digit Number puzzle.

MULTIPLYING A TWO-DIGIT NUMBER BY A SINGLE-DIGIT NUMBER

Directions: Using the digits 1 to 4 at most one time each, fill in the boxes to make the largest possible product.



2) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.
3) Collect students' interim answers without other students seeing/hearing those answers.
4) Select a common first answer where the thinking is only about putting the largest digit in the 10s place. For example, share this example with the class and show the multiplication using partial products.

   43 x 2 = 86

   40x2 + 3x2 = 80 + 6

5) Use another student example or pose this or a similar swap of two of the digits:

   42 x 3 = 126

   40x3 + 2x3 = 120 + 6

6) Pose questions to extend and connect student thinking.
   - Why is the answer so much larger when the 3 is the digit by itself and the 2 is part of the 42?
7) Continue to record student answers. Allow time for students to be satisfied that a product cannot be any larger and ask for reflections on what they learned.

   34 x 2 = 68 …. 30x2 + 4x2 = 60 + 8

   23 x 4 = 92 …. 20x4 + 3x4 = 80 + 12

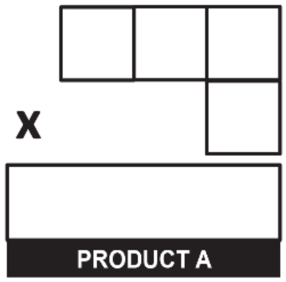
   32 x 4 = **128** … 30x4 + 4x2 = 120 + 8

   (**128** is the largest)

OPTIONAL HOMEWORK:

- Explain and offer a different kind of flash card. https://gfletchy.com/wp-content/uploads/2015/02/Multiplication-Subitizing-Cards-Dice.pdf
- How Grand is Your Total? Product A (4.NBT.5)

  Using the digits 0, 2, 4, and 6 once each, create the largest possible product.



420 x 6 = **2,520**

Lesson 7: More Multiplication

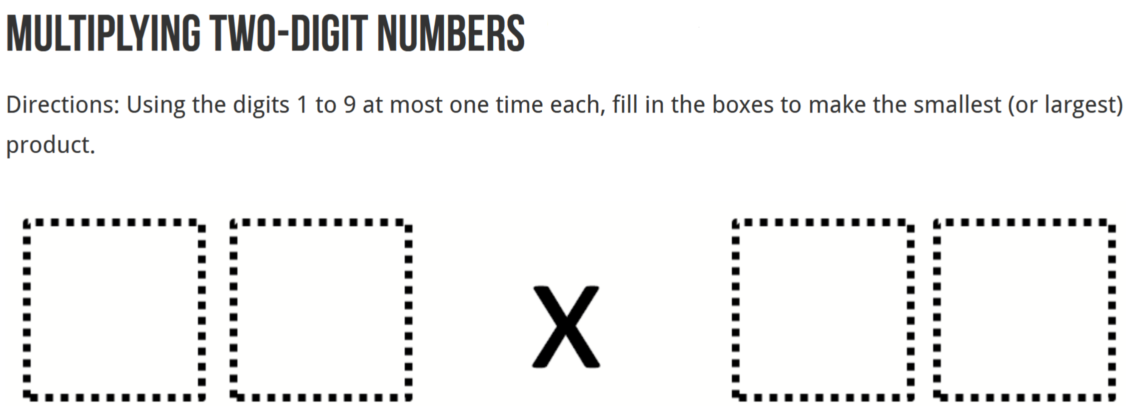
Lesson Objectives:

- *use place value and properties of operations to perform multi-digit arithmetic [Level B]*
- *continue to learn partial products [2.NBT.3]*
- *identify arithmetic patterns and explain them using properties of operations [3.OA.9]*
- *look for and make use of structure [MP.7]*
- *interpret products of whole numbers as the total number of objects in groups [3.OA.1]*
- *understand properties of multiplication [Level B]*

Multiplying Two-Digit Numbers (4.NBT.5 & 5.NBT.5)

Purpose: to deepen understanding of the power of place value within multiplication.

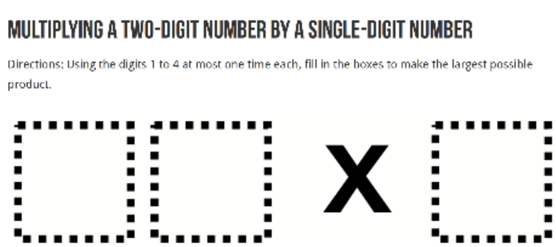
1) Display the Multiplying Two-Digit Numbers puzzle.



2) Offer options for student access, making note of the difference in directions.
   - Drag and drop calculator, includes whiteboard space. Directions as close to 1000 as possible: https://www.geogebra.org/m/mdhn4gzc
   - Drag and drop calculator. Directions as close to (a) 1000, (b) 2500, (c) 5000, (d) 7500 as possible: https://www.geogebra.org/m/ggpnh9br
3) During class, focus only on using this puzzle to make the smallest product.
4) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

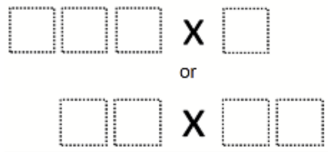
   Questions to move students along:

   - What was surprising about the problem from the last class? (simply putting the largest number in the biggest place value does not necessarily yield the largest product)



   - What digits would you select from the digits available (1 through 9)? Why?

- Which will yield the largest product? (2-digit x 2-digit)



- What does the number indicated by the two boxes on the left in the puzzle represent?
- What does the number indicated by the two boxes on the right in the puzzle represent?

5) After all students have had time to find a solution they are happy with, select a sequence for sharing student thinking. Doing the math puzzles yourself helps you understand different strategies. Consider putting together a sequence of math thinking that:
   - shows a shallow understanding of place value (initial thinking that is only about putting the largest digit in the 10s place), then shows
   - deeper understanding of the power of place value within multiplication, and finally
   - brings the deepest understanding of the power of place value within multiplication to find the smallest product.

   Smallest Product: 13 x 24 = 312

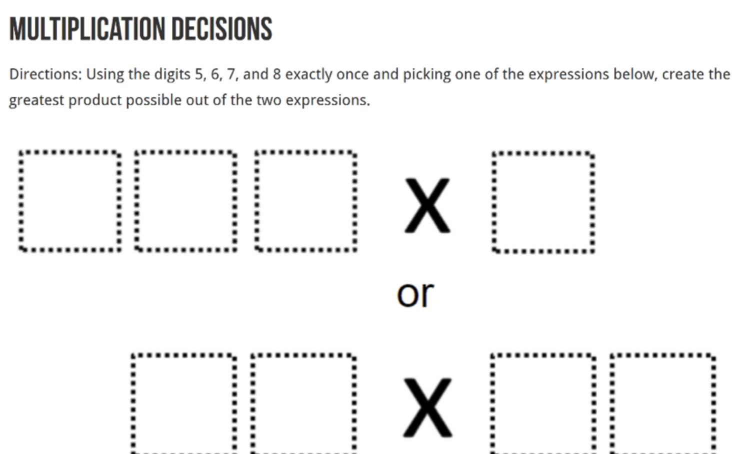
6) Ask students some reflection questions.
   - How would you read this problem? (collect suggestions until someone uses the language “sets of” or “groups of” or says something like “24 13s”)
   - How do you decide where to put the smallest digit?

OPTIONAL HOMEWORK:

- Use our class problem, Multiplying Two-Digit Numbers. (4.NBT.5) This time get the largest product possible.

  Largest Product: 96 x 87 = 8,352

- Multiplication Decisions (4.NBT.5)


MULTIPLICATION DECISIONS

Directions: Using the digits 5, 6, 7, and 8 exactly once and picking one of the expressions below, create the greatest product possible out of the two expressions.

❒❒❒ x ❒ =
765 x 8 = 6,120

❒❒ x ❒❒ =
85 x 76 = **6,460**

- Distributive Property with Four (4.NBT.6)

DISTRIBUTIVE PROPERTY WITH FOUR

Directions: Using the digits 0-9 at most once, create a true statement.

$$112 = (\square \times 4)+(\square \times 4)+(\square\square \times 4)$$

Any combination of 3 numbers that add to 28 in any order, including:

(1×4)+(3×4)+(24×4)

(2×4)+(7×4)+(19×4)

(3×4)+(4×4)+(21×4)

(4×4)+(5×4)+(19×4)

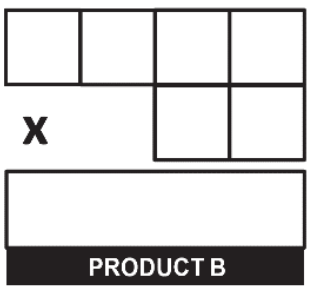
(5×4)+(6×4)+(17×4)

(6×4)+(7×4)+(15×4)

(7×4)+(8×4)+(13×4)

- How Grand is Your Total? Product B (4.NBT.5)

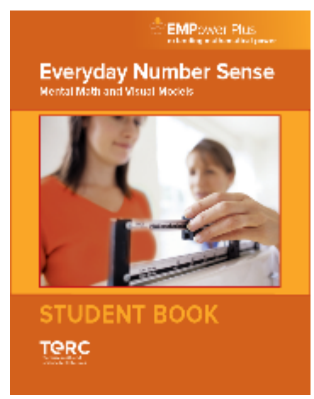
  Using the digits 0 through 5, make the largest possible product.



4310 x 52 = **224,120**

OPTIONAL HOMEWORK:

- EMPower's *Everyday Number Sense: Mental Math and Visual Models* student book Four Ways to Write Division (p. 209-210)



Lesson 8: Division

Lesson Objectives:

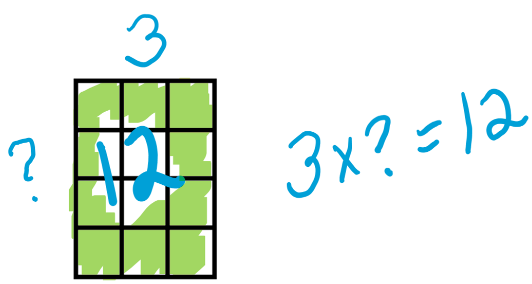
- *identify arithmetic patterns and explain them using properites of operations [3.OA.9]*
- *look for and express regularity in repeated reasoning [MP.8]*
- *interpret whole-number quotients using the quotitive, or measurement, model of division [3.OA.2]*
- *interpret whole-number quotients using the partitive, or sharing, model of division*
- *understand properties of multiplication and the relationship between multiplication and division [CCRS Level B]*

Draw Division (3.OA.2)

Purpose:

- *to support understanding of what division means.*
- *to connect division to multiplication.*

1) Ask students to represent 12 ÷ 3 using a drawing, or manipulatives such as tiles, pennies, scraps of paper, etc.
2) Make connections to students' previous multiplication drawings. Identify an example of division using area from student drawings or display an example of your own.



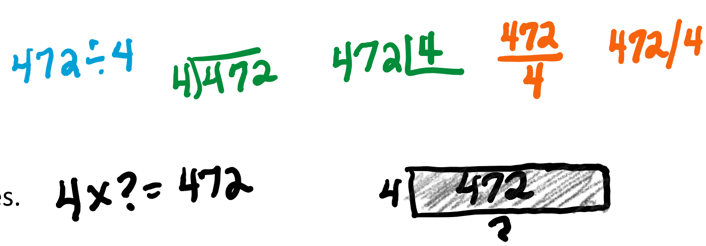
3) Use a pair of student drawings or verbal explanations to point out that there are two different ways to conceptualize division. [For teacher knowledge only, read more about partitive and quotitive models here: https://www.terc.edu/publications/two-ways-of-thinking-about-division]

Division Representations (3.OA.6)

Purpose:

- *to connect to students' prior experiences.*
- *to connect visual representations of division to symbolic representations.*

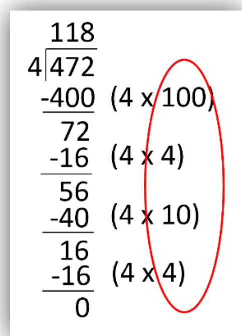
1) Ask students to share (and display) as many ways they can think of to notate division.
2) Share other ways to represent division, including notations used in different countries.



Partial Quotients (3.OA.2)

Purpose: to give students a less-rigid, more "forgiving" way to calculate which allows them to think about division concepts and place value.

[**Note:** To learn more about Partial Quotients, watch https://www.dropbox.com/s/55fwq9xpq029t2j/Partial_Quotient_Show.ppsx?dl=0.]



1) Model partial quotients, also referred to as the forgiving method. Do this multiple times in different places, as requested by students.
2) Make sure students have access to worked examples.

Desmos Classroom Activity: Greater, Less Than, or One

Purpose: to picture division and transfer that understanding to thinking about fractions as division.

[**Note:** To learn more about Desmos Classroom Activities, visit https://learn.desmos.com/activities-get-started]

1) Before class, "Assign" this classroom activity as a "single session code" or connect to Google Classroom https://teacher.desmos.com/activitybuilder/custom/60511b760eadba311f6b32d7
2) In Teacher Dashboard, pace to Screen 2 only.
3) Share the student code with students. Ensure students understand how to use the card sort.
4) Once students are finished with Screen 2, remove pacing. Ask students to share their understanding of when a division (or fraction) will be greater than one, less than one, or equal to one.

Whole Number Division (4.NBT.6)

Purpose:

- *to revisit ideas of equality*
- *to develop understanding of division*

WHOLE NUMBER DIVISION

Directions: Using the digits 1 to 9 at most one time each, fill in the boxes to make a true statement.

1) Display the Whole Number Division puzzle.
2) Ask students, "What do you notice?" to draw out observations about equality.
3) Allow students some time to think. This problem may take a long time for students because there are many things to remember at once:
   - equality
   - to use a single-digit in each box
   - to use a digit only once

   And, this problem has only one solution.

4) Select statements with equality issues such as 9 ÷ 1 = **9** ÷ 2 = 4.5 where the quotient to the first division is in the middle position. Spend time guiding students turning those into true statements without worrying about the puzzle rules (using the digits one time each, single digit in each box).

   9 ÷ 3 = 30 ÷ 10 = 3   9 ÷ 1 = 18 ÷ 2 = 9   9 ÷ 3 = 3 ÷ 1 = 3

   9 ÷ 3 = 12 ÷ 4 = 3   9 ÷ 3 = 6 ÷ 2 = 3

5) Give students more time to think, and space to manipulate numbers and discover strategies.

6) *Questions to move students along:*

   - What possible values can the single digit at the end have?
     - Why can't the end digit be 1? Explore.

       ❐ ÷ ❐ = 1   1 ÷ 1 = 1

       7 ÷ 7 = 1   2 ÷ 2 = 1

       6 ÷ 6 = 1   3 ÷ 3 = 1
     - Why can't the end digit be 3? Explore.

       Only 6 ÷ 2 = 3 and 9 ÷ 3 = 3, so there is no way to have an equation without repeating a digit.
     - Why can't the end digit be 4? Explore.

       Only 8 ÷ 2 = 4 and 4 ÷ 1 = 4, so there is no way to have an equation without repeating a digit.
     - Why can't the end digit be 5? Explore.

       10 ÷ 2 = 5, so there is no way to have an equation with only single digits with 5 as the dividend.
     - Why can't the end digit be 6? Explore.

       12 ÷ 2 = 6, so there is no way to have an equation with only single digits with 6 as the dividend.
     - Why can't the end digit be 7, 8, 9?

       All too large.
   - What are all the ways we can divide single digits and get 2?:

     8 ÷ 4 = 2   4 ÷ 2 = 2

     6 ÷ 3 = 2   2 ÷ 1 = 2

7) Only allow students to share the answer if everyone has discovered the answer.

   8 ÷ 4 = 6 ÷ 3 = 2

Partial Quotients

Purpose: to give students a less-rigid, more "forgiving" way to calculate which allows them to think about division concepts and place value.

1) Reteach partial quotients using 3⟌1536 to reinforce the ideas. Get input from students as you work through the problem. Show two different ways so students see the flexibility of partial quotients and build their understanding of division and place value:
   - one that uses a few large steps
   - one that uses more, small steps – such as mostly using repeats of three 100s

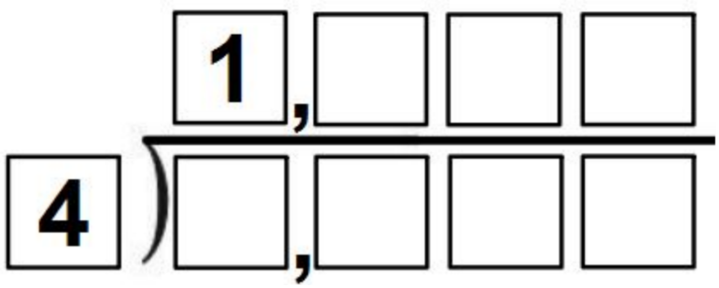
Division Fill in the Blanks (No Remainder) (4.NBT.6)

Purpose: to apply and extend previous learning to divide multi-digit by single-digit number.

1) Display the Division Fill in the Blanks (No Remainder) puzzle.

DIVISION FILL IN THE BLANKS (NO REMAINDER)

Directions: Fill in the blanks using all different non-zero digits (except the numbers 1 and 4, which have already been used) to make the greatest possible quotient.



2) If in class you have students from other countries and it hasn't come up yet, it may be helpful to point out that in other countries, the comma in 1,000 used to make large numbers easier to read would be a dot: 1.000. The reverse is true for the use of the decimal point. Students will see $1.00 in the United States when they might be used to writing it as $1,00 in their countries of origin.
3) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

   Questions to move students along:

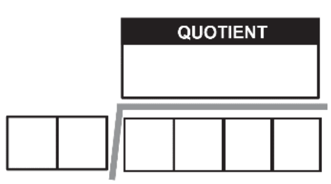
   - What do you know about the 4-digit number? (4 x 1-thousand-something = that number; it must be higher than 1,thousand-something; it must be even)
   - What number should ideally be placed in the hundreds place of the quotient? Why?
   - What can you determine about the units' digit of the dividend if the quotient has no remainder? (must be even)
4) Don't allow students to share the answer until everyone has the answer. Share homework suggestions with students who finish more quickly than others.

   Solution: 1,963 (the dividend is 7,852)

OPTIONAL HOMEWORK:

- How Grand is Your Total? Quotient

  Using the digits 0 through 9 one time each, make the largest possible quotient.



Solution: 9876 ÷ 01

NOTE: In the next class or between classes, reveal the How Grand is Your Total puzzle found in Lesson 11. Students will be able to use everything they've learned to find a Grand Total. Point out how much students have learned that will help in this puzzle, and show them what's left to learn before they solve the big puzzle. Ask some questions to stimulate thinking:

- Where would you "spend" your 9s?
- Where would you "spend" your 0s?
- If you don't need to use all these digits, what digits can you imagine not using? Why?

Lesson 9: Adding Fractions

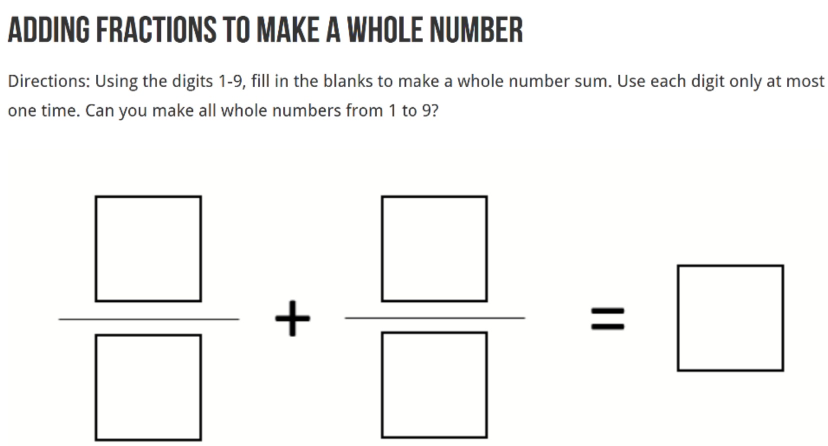
Lesson Objectives:

- *interpret fractions as division of the numerator and denominator [5.NF.3]*
- *look for and express regularity in repeated reasoning [MP.8]*
- *express whole numbers as fractions and recognize fractions that are equivalent to whole numbers [3.NF.3c]*

Adding Fractions to Make a Whole Number (5.NF.1)

Purpose: to connect fractions as division to addition of fractions.

- Display the Adding Fractions to Make a Whole Number puzzle.



- Focus students' attention on the equation (not instructions). One question at a time, ask students, "What do you notice?" and "What do you wonder?" Highlight student reasoning that shows algebraic thinking. Some concepts that may come up are connections to:
  - division,
  - whether the expected results of a single digit divided by a single digit are *greater than, less than, or equal to one* (found in the Desmos activity), and
  - properties of numbers – divide anything (except zero) by itself and the result is one.
- Consider problems that scaffold student understanding to be able to work on the puzzle, *Adding Fractions to Make a Whole Number*.
  - Make a fraction equal to:
    - 1 (answers: 1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 7/7, 8/8, 9/9)
    - 2 (answers: 2/1, 4/2, 6/3, 8/4)
    - 3 (answers: 3/1, 6/2, 9/3)
  - Ask, can we make any fractions less than one?
    - 1/2 (answers: 1/2, 2/4, 3/6, 4/8)
    - 1/3 (answers: 1/3, 2/6, 3/9)

    - Compare answers for fractions equal to 2 and fractions equal to 1/2. Compare answers for fractions equal to 3 and fractions equal to 1/3. What do you notice? Predict: Would this work for fractions equal to 4 and fractions equal to 1/4?
        - 4 (answers: 4/1, 8/2)
        - 1/4 (answers: 1/4, 2/8)
    - What fractions less than one can you add to equal one (if you didn't need to worry about using a digit twice)? (answers: 1/2 + 1/2 = 1; 1/4 + 3/4 = 1; 1/3 + 2/3 = 1)
- Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

  Questions to move students along:

    - When would the answer be greater than one? Less than one?
    - Ask a question about patterns of answers thus far. For example, why are all our answers higher than 4 so far?

      Two ideas students had that got them thinking outside their initial ideas:

        - We haven't been using fractions below 1 in our figuring
        - We haven't been using 2 ½ + ½ type fractions (such as 9/6 + 4/8)
- After all students have at least one correct solution, begin to compile answers where everyone can see. Ask about patterns.
- Allow additional time for students to find as many equations as they can.

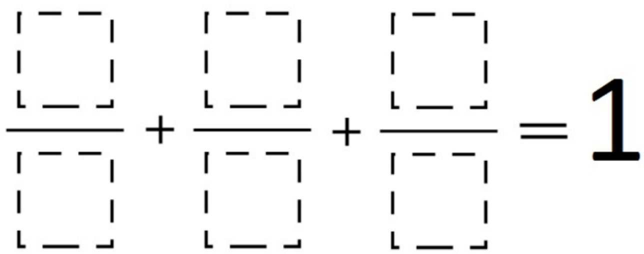
3/4 + 2/8 = 1	9/4 + 6/8 = 3	8/4 + 6/2 = 5	5/1 + 8/4 = 7	5/1 + 8/2 = 9
3/6 + 2/4 = 1	2/1 + 6/3 = 4	9/2 + 4/8 = 5	6/2 + 4/1 = 7	8/4 + 7/1 = 9
4/6 + 3/9 = 1	3/6 + 7/2 = 4	6/4 + 7/2 = 5	8/2 + 3/1 = 7	7/1 + 6/3 = 9
4/8 + 3/6 = 1	5/2 + 9/6 = 4	7/2 + 9/6 = 5	8/2 + 9/3 = 7	4/2 + 7/1 = 9
6/8 + 5/4 = 2	2/1 + 9/3 = 5	9/3 + 8/4 = 5	9/3 + 4/1 = 7	
6/9 + 4/3 = 2	3/1 + 8/4 = 5	9/3 + 4/2 = 5	6/2 + 5/1 = 8	
9/6 + 4/8 = 2	3/6 + 9/2 = 5	4/2 + 5/1 = 7	9/3 + 5/1 = 8	
5/2 + 4/8 = 3	4/2 + 3/1 = 5	5/1 + 6/3 = 7	4/2 + 6/1 = 8	

OPTIONAL HOMEWORK:

- Adding Fractions 4 (5.NF.1)

ADDING FRACTIONS 4

Directions: Using the integers 1 to 10 at most one time each, fill in the boxes so that the sum is equal to 1.



Some of the solutions:

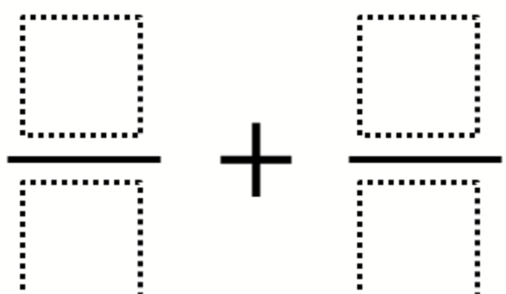
1/6 + 4/8 + 3/9

3/6 + 2/8 + 1/4

- Adding Fractions 3 (5.NF.1)

ADDING FRACTIONS 3

Directions: Using the digits 1 to 9 at most one time each, fill in the boxes so that the sum is as close to 1/2 as possible.



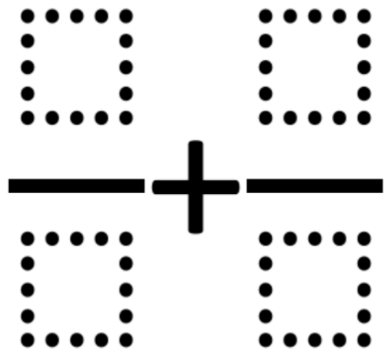
Two solutions:

2/8 + 1/4 and 3/9 + 1/6

- Adding Fractions (5.NF.1)

ADDING FRACTIONS

Directions: Using the digits 1 to 9 at most one time each, fill in the boxes to make the smallest (or largest) sum.



Largest: 9/1 + 8/2

Smallest: 2/9 + 1/8

Lesson 10: Multiplying Fractions

Lesson Objectives:

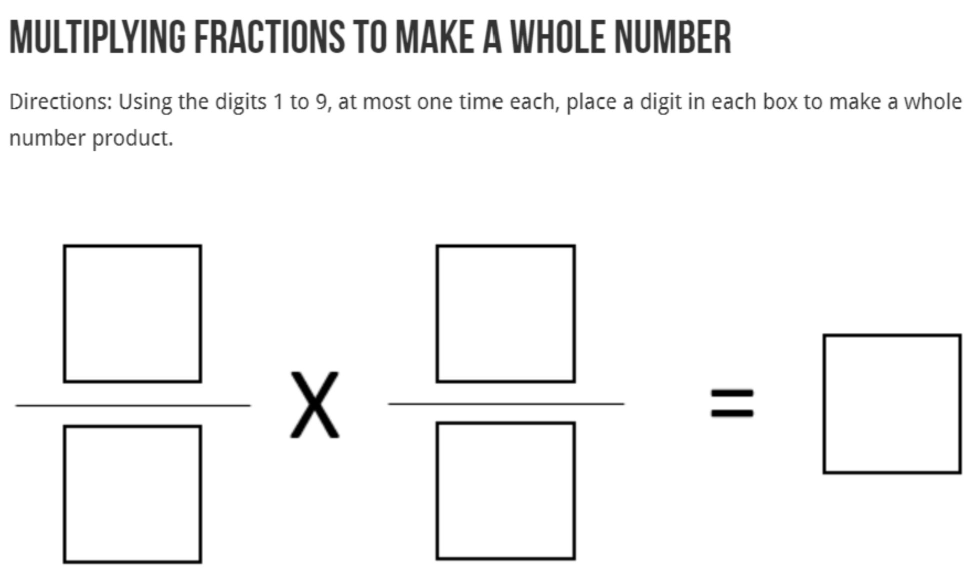
- *interpret fractions as division of the numerator and denominator [5.NF.3]*
- *look for and express regularity in repeated reasoning [MP.8]*
- *apply and extend previous understanding of multiplication and division to multiply fractions [Level C]*

Multiplying Fractions to Make a Whole Number (5.NF.4)

Purpose: to connect fractions as division to multiplication of fractions.

[**Note:** Pilot classes confirmed that students *do not* need a procedure for multiplying fractions in order to access this puzzle. Students drew on their knowledge of division notation and their knowledge of whole number multiplication (especially the *True or False?* activity in Lesson 6 where they learned to read multiplication as sets or groups, and transferred that knowledge to read ½ x 2 as “a half two times,” or “2 halves”). Students typically thought about this as two steps: create a whole number or benchmark fraction with each fraction, then multiply them.]

1) Display Multiplying Fractions to Make a Whole Number puzzle.



2) Ask students to look at the multiplying fractions boxes for the problem. One question at a time, ask students, “What do you notice?” “What do you wonder?” Highlight student reasoning that shows algebraic thinking. Some concepts that may come up are connections to:
   - division,
   - whether the expected results of a single digit ‘over’ a single digit are greater than, less than, or equal to one,
   - properties of numbers – divide anything (except zero) by itself and the result is one, and
   - whole number multiplication.

3) Ask for a prediction: Will the answer box (product) be larger or smaller than the value of each fraction? If it's helpful to them, students can try some fraction multiplication in their calculator to find out.

4) Allow students uninterrupted time to think, and space to manipulate numbers and discover strategies.

   Questions to move students along:

   - What have you noticed is NOT working? One class noticed:
     - We cannot make a fraction equal to 1 x □ (anything) because we can't use the same digit twice and the answer will be whatever was multiplied with the 1.
   - Think of a single-digit by single-digit multiplication problem that yields a single-digit answer. (2 x 3 = 6) Can you make this with fractions? (2/1 x 6/3 = 4)
   - Think of fractions that can be multiplied to make a single-digit answer. What equivalent fractions can you substitute so that you follow the rules of the puzzle?

     (1/2 x 2 = 1; 3/2 x 2/3 = 1)

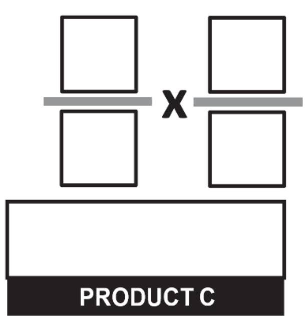
5) After all students have at least one correct solution, begin to compile answers where everyone can see. Ask about patterns.

6) Allow additional time for students to find as many equations as they can.

2/4 x 6/3 = 1	2/3 x 9/6 = 1	6/1 x 2/4 = 3	4/2 x 3/1 = 6
4/8 x 6/3 = 1	6/4 x 2/3 = 1	2/1 x 6/3 = 4	6/3 x 4/1 = 8
3/6 x 4/2 = 1	6/2 x 1/3 = 1	2/1 x 9/3 = 6	3/1 x 6/2 = 9
3/2 x 6/9 = 1	8/4 x 1/2 = 1	4/2 x 9/3 = 6	

OPTIONAL HOMEWORK:

- How Grand is Your Total? Product C
  Using the digits 0 through 9, one time each, make the largest possible product.



9/1 x 8/2 = 36 (same fractions as in the addition puzzle in Lesson 9)

9/2 x 8/1 = 36

Lesson 11: How Grand is Your Total?

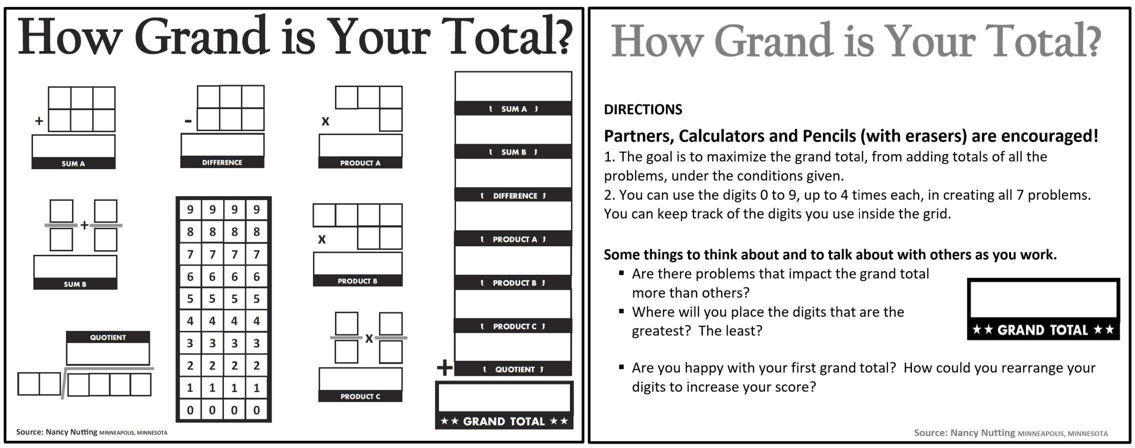
Lesson Objectives:

- *Make sense of problems and persevere in solving them. Mathematically proficient students start by … looking for entry points to its solution. They analyze givens, constraints, relationships, and goals… deeply understanding the power of place value in operations, and using properties of operations as a strategy to solve problems. (MP.1)*

How Grand is Your Total?

Purpose: to summarize the unit and work toward solving the ultimate Open Middle problem.

1) Share a copy of the How Grand is Your Total? puzzle and directions with students.



2) Ensure students understand the directions.
3) Because it is a big transition to switch from looking at individual problems to looking at the whole puzzle, ask students to make some whole-puzzle decisions first.
    - Where should we put our 9s? In what operation is a 9 most powerful? Where on the whole puzzle is the most powerful place?
    - Where should we put our 0s? Why should we use 0s if our goal is to get the largest Grand Total?
    - Where should we put our 1s? Where will a 1 be most powerful and make a number larger?
4) Give students time to work toward and each part of the puzzle and to determine a grand total.
5) Ask for a students' work and work as a class to rearrange digits to increase the total.
6) Give students time to rearrange their own puzzle and recalculate their grand total.
7) Ask students to summarize what they learned from this puzzle and during this unit. Consider keeping a list of students' contributions and posting it publicly.

We are not sharing our grandest total with you, and we suggest you don't share your own with students. The learning is in the doing! And redoing. And doing again.